Nucleon-alpha interaction to N2LO of Halo EFT

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The importance of a quantitative knowledge of $N-\alpha$ interaction is not only restricted to the basic two-body scattering process, but also extends to more general nuclear structural problems. Indeed processes involving α particles play an essential role in astrophysics, in particular in the context of big bang nucleosynthesis (BBN) and cosmology. The amount of hydrogen, helium and lithium produced within about three minutes from the Big Bang can be predicted through the baryonic matter density. The agreement between BBN and observations is good within experimental uncertainties, with the exception of a puzzling discrepancy for lithium, the so called lithium problem [1]. For this reason, a hot topic in modern cosmology and nuclear astrophysics is the shortage of the observed ^{7}Li in metal poor halo stars compared to the standard Big Bang prediction; a further problem concerns the isotopic ratio ^{6}Li over ^{7}Li, which is much larger than predictions. A motivational factor concerning this study is the synthesis of ^{6}Li via the $\alpha + d \longrightarrow ^6$ Li $+ \gamma$ reaction, which is an example of $N-N-\alpha$ interaction.

$N-\alpha$ interaction is also relevant for the study of nuclear halo states: they consist of a bound core surrounded by valence nucleons characterised by low separation energy, thus the nuclear size is much larger than the size of the bound core. This happens e.g. in the reaction $p +^7$ Be $\to^8$ B $+ \gamma$ which is important for solar neutrino production, where the nucleus ^{8}B is believed to be a two-body proton halo, ^{7}Be core and a proton; more complicated examples are ^{6}He and ^{11}Li, consisting of a bound core (^{4}He and ^{9}Li respectively) and two neutrons; they are called Borromean nuclei because they break into three fragments and never two, in analogy to Borromean rings.

The strong interaction between nucleons and α particles is in principle described by quantum chromodynamics (QCD), whose non-perturbative character prevents a viable calculational scheme. At sufficiently low energy the dynamical breakdown of (approximate) chiral symmetry allows to establish a systematic expansion in powers of small momenta to describe the interaction among nucleons as consisting of pion-exchanges and contact terms, which correspond to short-distance physics. At smaller energy even the pions can be integrated out and the interaction among nucleons reduces to contact terms only, resulting in the pionless effective field theory. In these frameworks the study of $N-\alpha$ scattering is effectively a 5-body problem. At even smaller energies, much smaller than the excitation energy of α particles, ∼20 MeV, the latters can be considered as structureless and an effective theory of interacting nucleons and αs can be formulated. The interaction among the active degrees of freedom consists of contact terms only, ordered in a derivative expansion. Correspondingly the $N-\alpha$ interaction potential in momentum space can be expanded in powers of momenta. After imposing all constraints from parity and time-reversal invariance, the strong potential in the center of mass frame ($\mathbf{p}_N + \mathbf{p}_\alpha = \mathbf{P} = 0$) takes the following minimal form up to the next-to-next-to leading order (N2LO)

$$\begin{aligned} v_{\rm s}(\mathbf{k},\mathbf{Q}) = & \overbrace{c_1}^{\rm LO} + \overbrace{c_2\mathbf{k^2} + ic_3\boldsymbol{\sigma}\cdot\mathbf{Q}\times\mathbf{k} + c_4\mathbf{Q}^2}^{\rm NLO} \\ & \underbrace{+ c_5\mathbf{k}^4 + c_6\mathbf{k}^2\mathbf{Q}^2 + c_7\mathbf{Q}^4 + c_8\left(\boldsymbol{\sigma}\cdot\mathbf{Q}\times\mathbf{k}\right)^2 + ic_9\left(\boldsymbol{\sigma}\cdot\mathbf{Q}\times\mathbf{k}\right)\mathbf{k}^2 + ic_{10}\left(\boldsymbol{\sigma}\cdot\mathbf{Q}\times\mathbf{k}\right)\mathbf{Q}^2}_{\rm N2LO}. \end{aligned} \tag{1}$$

Here $\mathbf{k} = \mathbf{p} - \mathbf{p}'$ is the relative momentum transfer, and $\mathbf{Q} = (\mathbf{p}+\mathbf{p}')/2$, with the relative momentum $\mathbf{p} = \mathbf{p}_N - \mathbf{p}_\alpha$ and primed quantities referring to final states. The contact interactions are smeared over a distance $a \sim 1/\Lambda$ by the introduction of a momentum cutoff, taken of a Gaussian form depending only on the relative momentum transfer $\mathbf{k}$,

$$F_\Lambda(\mathbf{k^2}) = \exp\left(-\frac{\mathbf{k}^2}{2\Lambda^2}\right). \tag{2}$$

In the spirit of Ref. [2], neglected high-momentum modes, like the excitation of degrees of freedom not explicitly retained in the effective theory, merely produce a shift in the low-energy constants (LECs) c_i's, which then become running coupling constants. Renormalization is carried on implicitly by fitting the LECs, for a given choice of Λ, to experimental observables. Once the LECs are fixed, predictions can be given for other observables. The truncation of the low-energy expansion to a given order introduces a cutoff dependence in the predictions, which can be taken as a measure of the importance of neglected orders, and thus of the theoretical uncertainty of the calculation. Provided the physical scales are well separated, within a retricted domain of energy, much smaller than the scale at which new physics starts to be relevant, the effective description maintains a certain degree of predictive power, but this can only be verified *a posteriori*. In the case at hand, reasonable values for Λ are below 200 MeV, since we must require $\Lambda^2/(2m_N) \lesssim 20$ MeV. The coordinate space expression of the above $N-\alpha$ interaction potential involves the Fourier transform of the cutoff function (2) and derivatives theoreof The resulting potential is

$$
\begin{aligned}
V_{\rm s}(\mathbf{r}) &= c_1 Z_\Lambda(r) - c_2 H_\Lambda(r) - c_3 Y_\Lambda(r) \mathbf{L}\cdot\mathbf{S} \\
&+ \frac{1}{4} c_4 \left[H_\Lambda(r) - 2\left\{ Z_\Lambda(r), \nabla^2 \right\} \right] + c_5 K_\Lambda(r) \\
&- \frac{1}{4} c_6 \left[K_\Lambda(r) - 2\left\{ H_\Lambda(r), \nabla^2 \right\} \right] + \frac{1}{8} c_7 \left[K_\Lambda(r) \right. \\
&\left. - \left\{ H_\Lambda(r), \nabla^2 \right\} + 2\left\{ Z_\Lambda(r), \nabla^4 \right\} + 4\nabla^2 Z_\Lambda(r) \nabla^2 \right] \\
&+ c_8 \left[\frac{4}{r} Y'_\Lambda(r) \left(\mathbf{L}\cdot\mathbf{S}\right)^2 + \frac{1}{2r} Z'''_\Lambda(r) - \left\{ Y_\Lambda(r), \nabla^2 \right\} \right] \\
&+ \left[\left(c_9 + \frac{1}{4} c_{10} \right) \frac{1}{r} Z'''_\Lambda(r) - \frac{1}{2} c_{10} \left\{ Y_\Lambda(r), \nabla^2 \right\} \right] \mathbf{L}\cdot\mathbf{S},
\end{aligned}
\tag{3}
$$

with $\mathbf{L}$ and $\mathbf{S}$ the orbital and spin angular momentum operators and the functions $Z_\Lambda(r)$, $H_\Lambda(r)$, $Y_\Lambda(r)$ and$K_\Lambda(r)$ are defined in Refs. [3,4]. Non-localities are generated already at NLO, corresponding to the appearance of $\mathbf{Q}^2$ terms in momentum space. At that order, sufficiently negative values of c_4 would result in a Hamiltonian unbounded from below. The problems are even more severe at N2LO, where the $\mathbf{Q}^4$ leads to the same pathology for all nonzero values of c_7. The origin of this ill behaviour is the fact that the adopted cutoff (2) only damps the momentum transfer, and not $\mathbf{Q}$. We can overcome this difficulty by adopting an additional cutoff directly in coordinate space, i.e. we modify the short-distance behaviour of the radial profile functions requiring that they go to zero for $r \to 0$. In practice we regulate the coordinate-space potential by an additional local cutoff,

$$
V_{\rm s}(\mathbf{r}) \to V_{\rm s}(\mathbf{r}) f_\Lambda(r), \tag{4}
$$

with $f_\Lambda(r)$ going to zero sufficiently fast for $r \ll 1/\Lambda$, similarly to what is done e.g. in Ref. [5]. In what follows we take

$$
f_\Lambda(r) = \left(1 - {\rm e}^{-\frac{r^2\Lambda^2}{2}} \right)^2, \tag{5}
$$

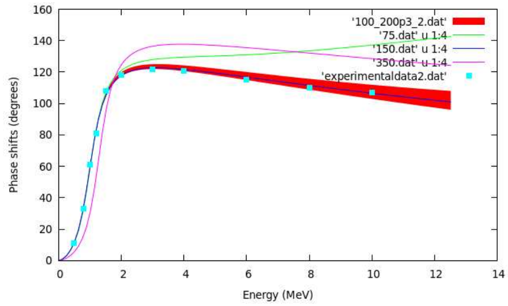

Figure 1. The $L_J = P_{3/2}$ phaseshift as obtained at NLO of the halo effetive theory. Only low-energy data are fitted, e.g. up to 1.5 MeV energy for $\Lambda = 100$ MeV.

which is enough, up to N2LO, to ensure a stable Hamiltonian for all $c_7 \geq 0$.

In addition to the strong interaction, considered to be isospin-symmetric, we have, in the case of protons, the Coulomb interaction, subjected to the same regulator (2). It then becomes

$$
V_{\rm C}(r) = 2\frac{\alpha_{\rm em}}{r} {\rm erf}\left(\frac{r\Lambda}{\sqrt{2}} \right) \tag{6}
$$

The scattering problem can be numerically solved using the complex Kohn variational principle, and the LECs can be fitted to the experimental low-energy phaseshifts. As an example, we show in the figure the resonant $P_{3/2}$ wave at NLO for different values of the cutoff Λ. The band indicates the theoretical uncertainty and is obtained by varying Λ between 100 and 200 MeV. By including also the N2LO it is possible to obtain a satisfactory fit without the emergence of spurious deeply bound states, which makes this model particularly well suited for systems with more particles, such as, e.g. the ^{6}He nucleus.

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