Pairing in spherical nuclei: self-consistent quasi-particle random phase approximation calculations with the Gogny interaction

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The description of the excited states of even-even open shell nuclei requires an extension of the Random Phase Approximation (RPA) theory, called quasi-particle RPA (QRPA) [1], built to handle pairing and partial occupation probabilities of the single particle (s.p.) levels. In Ref. [2] we constructed a QRPA model on top of our HF+BCS approach [3, 4, 5] by using the same finite-range interaction of Gogny type that provides stability of the pairing results [6]. Our calculations are fully self-consistent: the same interaction is used to generate s.p. levels, pairing and QRPA excitations.

In the QRPA theory, an excited state $|k\rangle$ of angular momentum J, third component M, parity Π, and excitation energy ω_k, is generated by the $Q^\dagger$ operator

$$Q^\dagger_k = \sum_{\mu\le\mu'} \Big[X^k_{\mu\mu'} A^\dagger_{\mu\mu'}(k) + (-1)^{J+M+1} Y^k_{\mu\mu'} A_{\mu\mu'}(k) \Big], \qquad (1)$$

where $A^\dagger_{\mu\mu'}(k)$ and $A_{\mu\mu'}(k)$ are the quasi-particle pair creation and annihilation operators. By using standard techniques the QRPA secular equations can be written as

$$\begin{bmatrix} \mathcal{A} & \mathcal{B} \\ \mathcal{B} & \mathcal{A} \end{bmatrix} \begin{bmatrix} X^k \\ -Y^k \end{bmatrix} = \omega_k \begin{bmatrix} X^k \\ -Y^k \end{bmatrix}, \qquad (2)$$

where the $\mathcal{A}$ and $\mathcal{B}$ contain both particle-hole (F) and particle-particle (G) interaction matrix elements.

We have investigated the excitation spectrum of 6 oxygen isotopes, from $A = 16$ to $A = 26$, 12 calcium isotopes, from $A = 40$ to $A = 62$, and 9 isotones with $N = 20$, from ^{30}Ne to ^{46}Fe by using a HF+BCS+QRPA approach with the aim of studying global effects of the pairing. In our approach, the pairing is active in the construction of the s.p. configuration space, generated by HF+BCS calculations, and in the QRPA results in the G terms. We carried out calculations without G terms using HF s.p. states (RPA) and HF+BCS s.p. states (QRPA(F)) and full HF+BCS+QRPA calculations (QRPA), to disentangle the source of the pairing effects. We summarize here below the main results of our study.

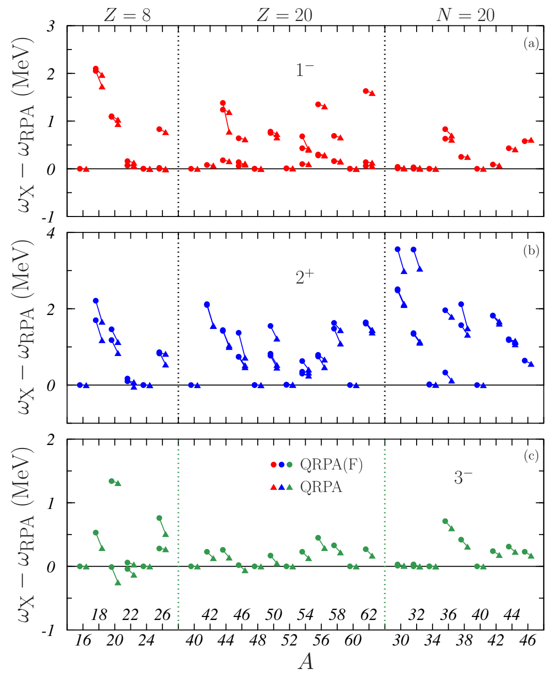

Figure 1. Differences between the excitation energies obtained in QRPA(F) (circles) and QRPA (triangles) and those found in RPA calculations, for the 1^-, panel (a), 2^+, panel (b), and 3^-, panel (c), excited states.

Firstly, we investigated how the pairing modifies the occupation numbers and the energies of the s.p. states that are input of the QRPA calculations. We found that these two quantities change in uncorrelated way.

We analysed the effect of the pairing on the low-lying excitations and on the giant resonance regions. As example of the first study, we show in Fig. 1, for the principal states of 1^-, 2^+ and 3^- multipolarities, $\omega_{\rm QRPA(F)} - \omega_{\rm RPA}$ (solid circles) and $\omega_{\rm QRPA} - \omega_{\rm RPA}$ (triangles) differences.

In the great majority of the cases considered, the QRPA(F) and QRPA excitation energies are larger than the energies obtained in RPA calculations. The differences between QRPA and QRPA(F) calculations are only due to the presence of the pairing force in the G terms of the QRPA. The results indicate that, in general, $\omega_{\mathrm{QRPA(F)}}$ is larger than ω_{QRPA}.

On the other hand we studied how the pairing modifies the collectivity of the excited states, founding that the pairing generates more collectivity in the 2^+ excitations. The opposite effect is present in the 2^- excitation. For other multipoles the situation is more confused and each state has to be investigated individually.

We have analysed the low-energy excited states dominated by a coniugate quasi-particle configuration implying a s.p. transition between a partially occupied state and itself. They play an important role in those calculations where the pairing interaction is active. In Fig. 2 we present energies (left panels) and $B(E2)$ values (right panels) of the lowest QRPA(F)and QRPA 2^+ states, most of them dominated by conjugate configurations. Our results, indicated by the solid circles and triangles, are compared to the experimental data (solid squares)[8],[9].

All the QRPA(F)energies are larger than those obtained in QRPA calculations. For the oxygen isotopes, the agreement between our calculations and those of Ref. [10] is good, better of the description of the experimental data. The relativistic calculations of [13, 14] show behaviors analogous to ours. As far as calcium isotopes are concerned, our results are in agreement with those of Ref. [11] in both energy and $B(E2)$ values, with the exception of the ^{40}Ca nucleus. In general, we predict excitation energies larger than those experimentally found, and $B(E2)$ values smaller than the observed ones. The calculations of Ref. [12] generate smaller values of the excitation energies and very large $B(E2)$ values, even larger than those measured. For $N = 20$ isotones, our results describe reasonably well the experimental data for ^{34}Si, ^{36}S and ^{38}Ar, the discrepancies for the ^{30}Ne and ^{32}Mg nuclei are remarkable. By using a strong pairing interaction the results of Ref. [9] are able to reproduce the small experimental energies and the large $B(E2)$ values.

As far as the giant resonance regions are concerned, we have analysed the centroid energy of all the responses of all the case studied. We found that, in general the pairing enhances the position of the centroid. The greater differences are for QRPA(F) calculations. The larger effects are for the $N = 20$ isotones and for the 1^- excitation.

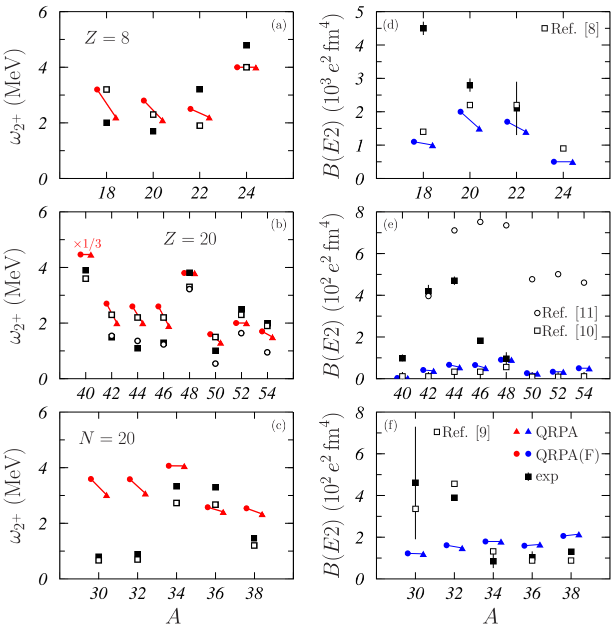

Figure 2. Energies (left panels) and B-values (right panels) of the lowest 2^+ excitations for some of the nuclei studied. The solid circles and triangles show our QRPA(F) and QRPA results, respectively. The solid squares indicate the experimental data from the compilation of Ref. [8], with the exception of the $B(E2)$ value of ^{30}Ne taken from Ref. [9]. The open squares represent the results of Refs. [10], for $Z = 8$, [11], for $Z = 20$, and [9], for $N = 20$. The open circles show the results of Ref. [12] also for $Z = 20$. Our QRPA and QRPA(F) energy values for the ^{40}Ca nucleus are divided by a factor 3.

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