Three-dimensional natural almost contact structures

Giovanni Calvaruso [1] and Antonella Perrone [1]

[1]Dipartimento di Matematica e Fisica, Università del Salento, Italy

Almost contact metric structures are the natural odd-dimensional analogue to almost Hermitian structures. In recent years, almost contact metric geometry has been intensively investigated, also in relation with CR structures.

The more relevant and well understood class of almost contact metric structures, is given by the normal ones. As it is well known, an almost contact structure (φ, ξ, η) is *normal* if and only if

$$[\varphi, \varphi] + 2d\eta \otimes \xi = 0, \tag{1}$$

where $[\varphi, \varphi]$ is the Nijenhuis torsion tensor of φ. It is evident that the above Equation (1 holds independently by any Riemannian metric compatible with the structure, although it has several consequences on the Levi-Civita connection and curvature of any of such a metrics. We now introduce the following definition.

Definition An almost contact structure (φ, ξ, η) is said to be *natural* if $\xi \in \mathrm{Ker} d\eta$.

Also in the case of natural contact metric structures, the defining property is independent of the particular Riemannian metric compatible with the structure. At the same time, this property influences the geometry of any compatible metric. After showing that natural almost contact metric structures generalize both normal structures and contact metric structures, we investigated their geometric properties, with particular regard to the three-dimensional case. We then classified left-invariant natural almost paracontact metric structures on three-dimensional Lie groups and related classes of examples. Moreover, we also classified CR structures corresponding to three-dimensional left-invariant natural almost contact structures and determined the minimal ones.

REFERENCES

1. G. Calvaruso and Antonella Perrone, *Natural almost contact structures and their 3D homogeneous models*, Math. Nachr. **289** (2016), 1370–1385.