On the geometry of Hamiltonian formalism for partial differential equations

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The integrability of Partial Differential Equations (PDEs) is quite often obtained from the existence of distinguished algebraic and/or geometric structures. More particularly, a feature of many (non-linear) integrable differential equations is the existence of an infinite sequence ('hierarchy') of commuting higher (or generalized) symmetries and/or conservation laws. Such sequences are often generated through differential operators.

We recognised that a certain class of third-order homogeneous operators can be identified with quadratic line complexes, which are algebraic varieties in the Grassmannian of all lines of a projective space [**?**, **?**]. The results have been exposed in several universities and conferences in 2013, 2014, 2015 and 2016 (for this last year only see [**?**, **?**, **?**]). The results have been used in a recent paper [**?**].

Recently, a huge number of inequivalent integrable systems have been discovered through the above classifications. The systems are a straightforward bi-Hamiltonian generalization of the famous Korteweg-de Vries equation [**?**].

R. Vitolo developed the program CDE that is a package of the Reduce Computer Algebra System (which is now free software) [**?**]. This is one of the few publicly available programs which is able to make computations on integrability differential operators. The theory on which the software is based is exposed in a series of papers [**?**, **?**]. The experience and results that have been obtained so far will be collected in a forthcoming book [**?**].

R. Vitolo now is one of REDUCE developers. In collaboration with A.C. Norman (Trinity College, Cambridge) he wrote an introduction to Reduce programming [**?**], which is now part of the official Reduce documentation, see `http://sourceforge.net/projects/reduce-algebra/`.

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