Dynamic Taxes for Polynomial Congestion Games

V. Bilò [1] and C. Vinci [2]

[1]Dipartimento di Matematica e Fisica, Università del Salento, Italy

[2]Gran Sasso Science Institute, L'Aquila, Italy

Many applications in real-life systems – such as routing on transportation or computer networks, machine scheduling, resource sharing, group formation – are suitably modeled by congestion games [8]. In these games, there is a set of non-cooperative selfish players sharing a set of *resources* and each resource incurs a certain *latency* to the players using it. Each player has an available set of strategies, where each strategy is a non-empty subset of resources, and aims at choosing a strategy minimizing her cost which is defined as the sum of the latencies experienced on all the selected resources.

It is well-known that selfish behavior produces suboptimal outcomes and several metrics, such as the price of anarchy and the price of stability, have been introduced to measure this phenomenon. In such a setting, it becomes natural to ask whether or nor the impact of selfish behavior on the system efficiency can be mitigated by an external mild intervention. Toward this end, some effective approaches – namely taxation [5], Stackelberg strategies [7,4], coordination mechanisms [6] – have been fruitfully proposed and analyzed in the literature of congestion games. The first approach aims at discouraging the use of certain (usually highly congested) resources by forcing players to pay taxes for using them, the second one assumes that a central authority has the power of controlling and dictating the behavior of a fraction of the players, while coordination mechanisms partially redefine the resource latency functions according to a certain predetermined policy.

In this work, we extend and generalize the results of [5] along two orthogonal directions by determining efficient tax functions for congestion games with polynomial latencies with respect to a variety of different solution concepts ranging from approximate pure Nash equilibria up to approximate coarse correlated equilibria, and including also approximate one-round walks starting from the empty state.

The technique we exploit to derive our results is based on a non-trivial application of the primal-dual method introduced by [2]. An important feature of this method, as shown in [3], is that any upper bound derived through its application which holds for ϵ-approximate pure Nash equilibria directly extends to ϵ-approximate coarse correlated equilibria. For such a reason, although throughout the paper for the sake of simplicity we shall only refer to ϵ-approximate pure Nash equilibria, all the proposed upper bounds have to be intended to hold for ϵ-approximate mixed Nash equilibria, ϵ-approximate correlated equilibria and ϵ-approximate coarse correlated equilibria as well. We stress that achieving an exact characterization of the worst-case performance guarantee of approximate (coarse) correlated equilibria is of fundamental importance in light of the recent negative result by [1] stating that computing approximate (coarse) correlated equilibria optimizing the social welfare is $\mathcal{NP}$-hard in many games of interest.

Not surprisingly, according with the achievements of [5], the most interesting results are obtained for the case of refundable taxes. For either weighted and unweighted games, we determine $\mathcal{T}_{d+1}(1+\epsilon)$-efficient taxes based on a social optimum with respect to ϵ-approximate equilibria and to both the total latency and the weighted total latency, where $\mathcal{T}_i(x)$ denotes the ith Touchard polynomial. We also show that this is the best possible efficiency achievable by a tax function based on a social optimum even in unweighted games. From a computational point of view, we show how to compute in polynomial time $(1+\beta)\mathcal{T}_{d+1}(1+\epsilon)$-efficient taxes in unweighted games, for any arbitrary $\beta > 0$. Still for unweighted games, we determine $\mathcal{G}_{d+1}(1+\epsilon)$-efficient taxes based on a social optimum with respect to ϵ-approximate one-round walks starting from the empty state and show that this is the best possible efficiency achievable by a tax function based on a social optimum, where $\mathcal{G}_i(x)$ denotes the ith Geometric polynomial. Also in this case, we show how to compute in polynomial time $(1+\beta)\mathcal{G}_{d+1}(1+\epsilon)$-efficient taxes, for any arbitrary $\beta > 0$. Interestingly, our upper bounds are derived by exploiting the combinatorial definition of the polynomials $\mathcal{T}_i(x)$ and $\mathcal{G}_i(x)$, while our lower bounds are constructed by relying on their analytical characterization. Finally, for weighted games, we determine efficient taxes based on a social optimum with respect to ϵ-approximate one-round walks starting from the empty state and to the weighted total latency. For non-refundable taxes and weighted games, we determine efficient taxes based on a social optimum with respect to ϵ-approximate equilibria and to the total cost and efficient taxes based on a social optimum with respect to ϵ-approximate one-round walk starting from the empty state and to the weighted total cost.

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