Regular subgroups of the affine group and asymmetric product of radical braces

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Let V be a vector space over a field F. Clearly, the group $T(V)$ of all the translations of V and any of its coniugate subgroups by an element of $GL(V)$ are abelian regular subgroups of the affine group $AGL(V)$. It is well known that if V is finite-dimensional, then any abelian regular subgroup of $AGL(V)$ intersects nontrivially the group of translations. This does not necessarily happen when V is infinite-dimensional, as has been pointed out by Caranti, Dalla Volta and Sala [2]. In [7], Hegedűs constructed interesting examples of nonabelian regular subgroups of some affine groups over a finite-dimensional vector space containing no nontrivial translation. The interest for these special subgroups originated from the seminal paper by Liebeck, Praeger and Saxl [8]. Currently other examples of this type can be found, for instance, in [2], [3], and [13].

Caranti, Dalla Volta and Sala [2] obtained a simple description of the abelian regular subgroups of the affine group $AGL(V)$ in terms of commutative radical algebras with the underlying vector space V. Afterwards a description of all regular subgroups, not necessarily abelian, of an affine group was obtained by Rizzo and the first author [4] in terms of radical braces over a field, a generalization of radical algebras. These new structures, introduced by Rump in [10], are very closely related to non-degenerate involutive set-theoretic solutions of the quantum Yang-Baxter equation. Thus the open problem of determining all regular subgroups of an affine group $AGL(V)$, formulated in [9], may be replaced by that of determining all radical braces with the underlying vector space V.

The aim of this paper is to introduce the asymmetric product of radical braces, a construction which extends the semidirect product of radical braces introduced in [11] and rewritten in [5]. This new construction allows to obtain rather systematic constructions of regular subgroups of the affine group and, in particular, this approach allows to put the regular subgroups constructed by Hegedűs in [7] in a more general context.

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