

On the geometry of Hamiltonian formalism for partial differential equations

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The integrability of Partial Differential Equations (PDEs) is quite often obtained from the existence of distinguished algebraic and/or geometric structures. More particularly, a feature of many (non-linear) integrable differential equations is the existence of an infinite sequence (‘hierarchy’) of commuting higher (or generalized) symmetries and/or conservation laws. Such sequences are often generated through differential operators.

We recognised that a certain class of third-order homogeneous operators can be identified with quadratic line complexes, which are algebraic varieties in the Grassmannian of all lines of a projective space [2, 3]. The results have been exposed in several universities and conferences in 2013, 2014 and 2015 (for this last year only see [7, 8, 9, 10, 11, 12]). The results have been used in a recent paper [4].

R. Vitolo developed the program CDE that is a package of the Reduce Computer Algebra System (which is now free software) [13]. This is one of the few publicly available programs which is able to make computations on integrability differential operators. The theory on which the software is based is exposed in a series of papers [5, 6].

R. Vitolo now is one of REDUCE developers. In collaboration with A.C. Norman (Trinity College, Cambridge) he wrote an introduction to Reduce programming [14], which is now part of the official Reduce documentation, see <http://sourceforge.net/projects/reduce-algebra/>.

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10. R. Vitolo, *On the classification of third-order homogeneous Hamiltonian operators* poster at the conference Physics and Mathematics of Nonlinear Phenomena, Gallipoli, June 2015.
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12. R. Vitolo, *On the geometry of third-order homogeneous Hamiltonian operators* seminar at the University of Leeds, November 2015.

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