On the Price of Stability of Fractional Hedonic Games

V. Bilò [1] and A. Fanelli [2] and M. Flammini [3] and G. Monaco [3] and L. Moscardelli [4]

[1]Dipartimento di Matematica e Fisica, Università del Salento, Italy

[2]CNRS, (UMR-6211), France

[3]Università di L'Aquila, Italy

[4]Università di Chieti-Pescara, Italy

In many economical, social and political situations, individuals carry out activities in groups rather than alone and on their own. In these scenarios, understanding the "happiness" of each member of the group becomes of crucial importance. As examples, the utility of an individual in a group sharing a resource depends both on the consumption level of the resource and on the identity of the members in the group; similarly, the utility for a party belonging to a political coalition depends both on the party trait and on the identity of its members.

Hedonic games, introduced in [3], describe the dependence of a player's utility (or payoff) on the identity of the members of her group. They are games in which players have preferences over the set of all possible player partitions (called clusterings). In particular, the utility of each player only depends on the composition or structure of the cluster she belongs to. In the literature, a significant stream of research considered this topic from a strategic cooperative point of view, with the purpose of characterizing the existence and the properties of coalitional structures such as, for instance, the core. Nevertheless, studying strategic solutions under a non-cooperative scenario (such as, for instance, Nash equilibria) becomes of fundamental importance when considering huge environments (like the Internet) lacking a social planner or where the cost of coordination is tremendously high.

In this work, we consider the class of (symmetric) *fractional hedonic games* introduced in [2]. These games are defined by a graph in which nodes represent players and the weight of each edge measures the happiness of its two incident players when belonging to the same cluster. The utility that player i gets when belonging to cluster C is given by the total weight of edges which are incident to i and to some other player belonging to C (the total happiness of i in C) divided by the cardinality of C, i.e., the number of its nodes. The social welfare of a clustering is the sum of the players' utilities.

Fractional hedonic games model natural behavioral dynamics in social environments that are not captured by additive separable ones, that is, games in which the utility of a player is simply defined as her total happiness. In particular, fractional hedonic games defined on ***undirected and unweighted bipartite graphs*** suitably model a basic economic scenario in which each player can be considered as a buyer or a seller. There are only edges connecting buyers and sellers and every player sees a player of the same type as a market competitor. In a situation of free movement, each player prefers to be situated in a group (market) with a small number of competitors: Each buyer wants to be situated in a group with many sellers and few other buyers, thus maximizing their ratio, in order to decrease the price of the good. On the other hand, a seller wants to be situated in a group maximizing the number of buyers against the number of sellers, in order to be able to increase the price of the good and gain a higher profit. This senario is referred to in [2] as *Bakers and Millers* and can be generalized to situations in which there are more than two types of players by means of k-partite graphs.

In this setting, a clustering is *Nash stable* (or it is a *Nash equilibrium*) if no player can improve her utility by unilaterally changing her own cluster. Our aim is to understand the performance of Nash stable clusterings. In particular, we study the quality of a best Nash stable outcome and refer to the ratio of its social welfare to the one of the socially optimal clustering as to the *price of stability* (a study on the price of stability for multi-agent systems can be found in [4]). A best Nash stable outcome has a natural meaning of stability, since it is an optimal solution among the ones which can be accepted by selfish players [1]. Moreover, in many networking applications and multi-agent systems, agents are never completely unrestricted; rather, they interact with an underlying protocol that essentially proposes a collective solution to all participants, each of whom can either accept it or defect from it. As a result, it is in the interest of the protocol designer to seek for a best solution at equilibrium. In fact, this can naturally be viewed as the optimum subject to the constraint that the solution has to be stable, with no agent having an incentive to unilaterally defect from it once it is offered.

Our Contribution. In this paper, we focus on the price of stability of fractional hedonic games played on undirected and unweighted graphs. For general graph topologies, we give a lower bound of 2. Moreover, we provide an upper bound of 4 which holds under the assumption that the game possesses a 2-Strong Nash stable clustering, that is, a clustering such that no pair of players can improve their utility by simultaneously changing her own cluster. However, we show that there are games for which such a condition is not always guaranteed. We then focus on games played on specific graph topologies. In particular, for triangle-free graphs, we prove an upper bound of 4, while, for bipartite graphs, we give an upper bound of $6(3-2\sqrt{2}) \approx 1.0294$ and a lower bound of 1.003. We stress that our upper bounds on the price of stability directly extend also to the utility function considered by Olsen in [5].

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