Precision measurement of three neutrino oscillation parameters

Daniele Montanino [1]

[1]Dipartimento di Matematica e Fisica "Ennio de Giorgi", Università del Salento and INFN, sez. di Lecce

Neutrino oscillations are a well-estabilished quantum phenomenon in which neutrinos can change their flavor during propagation. The origin of this phenomenon comes from the fact that flavor (i.e., interaction) eigenstates are not the same of mass (i.e., propagation) eigenstates. Instead, the two basis are related by a unitary matrix

$$\left(\begin{array}{c}\hat{\nu}_e\\ \hat{\nu}_\mu\\ \hat{\nu}_\tau\end{array}\right) = \mathbf{U}\cdot\left(\begin{array}{c}\nu_1\\ \nu_2\\ \nu_3\end{array}\right)\,. \quad (1)$$

Here $\hat{\nu}_{e,\mu,\tau}$ are the "interaction" eigenstates (which convert into the corresponding leptons in Charge Current interactions), ν_i the mass eigenstates with mass m_i. By convention $m_1 < m_2 \ll m_3$ (Normal Hierarchy, NH) or $m_3 \ll m_1 < m_2$ (Inverted Hierarchy, IH). Since oscillations depend only to $\delta m^2_{jk} = m^2_k - m^2_j$ oscillation experiments can probe only two mass square differences and not absolute neutrino masses. By convention we choose $\delta m^2 = m^2_2 - m^2_1$ and $\Delta m^2 = m^2_3 - (m^2_1 + m^2_2)/2$ as free parameters. The sign of Δm^2 determines the hierarchy ("+" for NH, "−" for IH). This is schematically shown in Fig. 1. Although mass square differences are well known, the true hierarchy is still unknown.

The unitary matrix $\mathbf{U}$ can be parameterized as the product of three unitary matrices $\mathbf{W}^{ij}$

$$\mathbf{U} = \mathbf{W}^{23}(\theta_{23})\cdot\mathbf{W}^{13}(\theta_{13},\delta)\cdot\mathbf{W}^{12}(\theta_{12})\,, \quad (2)$$

with

$$\begin{aligned}\mathbf{W}^{23}(\theta_{23}) &= \left(\begin{array}{ccc}1 & 0 & 0\\ 0 & C_{23} & S_{23}\\ 0 & -S_{23} & C_{23}\end{array}\right)\\ \mathbf{W}^{13}(\theta_{13},\delta) &= \left(\begin{array}{ccc}C_{13} & 0 & S_{13}e^{i\delta}\\ 0 & 1 & 0\\ -S_{13}e^{-i\delta} & 0 & C_{13}\end{array}\right)\\ \mathbf{W}^{12}(\theta_{12}) &= \left(\begin{array}{ccc}C_{12} & S_{12} & 0\\ -S_{12} & C_{12} & 0\\ 0 & 0 & 1\end{array}\right)\,, \quad (3)\end{aligned}$$

where we have used the shorthand $C_{ij} \equiv \cos\theta_{ij}$ and $S_{ij} \equiv \sin\theta_{ij}$ with $\theta_{ij} \in [0,\pi/2]$ are three mixing angles and $\delta \in [0, 2\pi]$ is a phase related to CP violation in the leptonic sector.

A neutrino $\hat{\nu}_\alpha$ is emitted in interaction eigenstate and thus is a linear combination of mass (propagation) eigenstates ν_i. After that the ν_i's propagate with different de Broglie wavelength: at detection the final state is in general a different combination of the initial state. As a consequence, there is a nonzero probability to detect the neutrino in a different interaction eigenstate $\hat{\nu}_\beta$, $\beta \neq \alpha$. The conversion probability at distance x from source for a neutrino with energy E_ν is given by

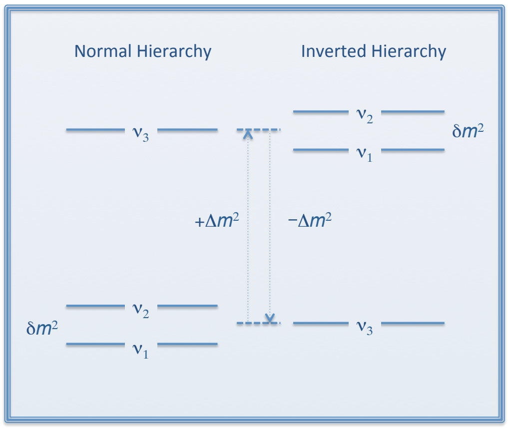

Figure 1. Schematic representation of possible mass neutrino hierarchies.

$$\begin{aligned}P_{\alpha\beta}(x) &= \delta_{\alpha\beta}\\ &- 4\sum_{k>j}\Re[U_{\alpha_j}U^*_{\beta j}U^*_{\alpha k}U_{\beta k}]\sin^2\left(\frac{\phi_{jk}}{2}\right)\\ &+ 2\sum_{k>j}\Im[U_{\alpha_j}U^*_{\beta j}U^*_{\alpha k}U_{\beta k}]\sin\phi_{jk}\,, \quad (4)\end{aligned}$$

with

$$\phi_{jk} = \frac{\delta m^2_{jk}}{4E_\nu}x\,. \quad (5)$$

In particular, the effect of the phase δ on neutrino oscillations is that $P_{\alpha\beta} \neq P_{\bar{\alpha}\bar{\beta}}$ (CP violation effect) or $P_{\alpha\beta} \neq P_{\beta\alpha}$ (T violation effect) unless $\delta = 0, \pi$. For simplicity, in the following we refer simply to CP violations.

We mention also that for neutrinos traveling in matter (such as Solar neutrinos) the interaction with the background matter modifies substantially the oscillation probability (the so called Mikheyev-Smirnov-Wolfenstein effect). We do not discuss here this effect since it would be beyond the scope of this report.

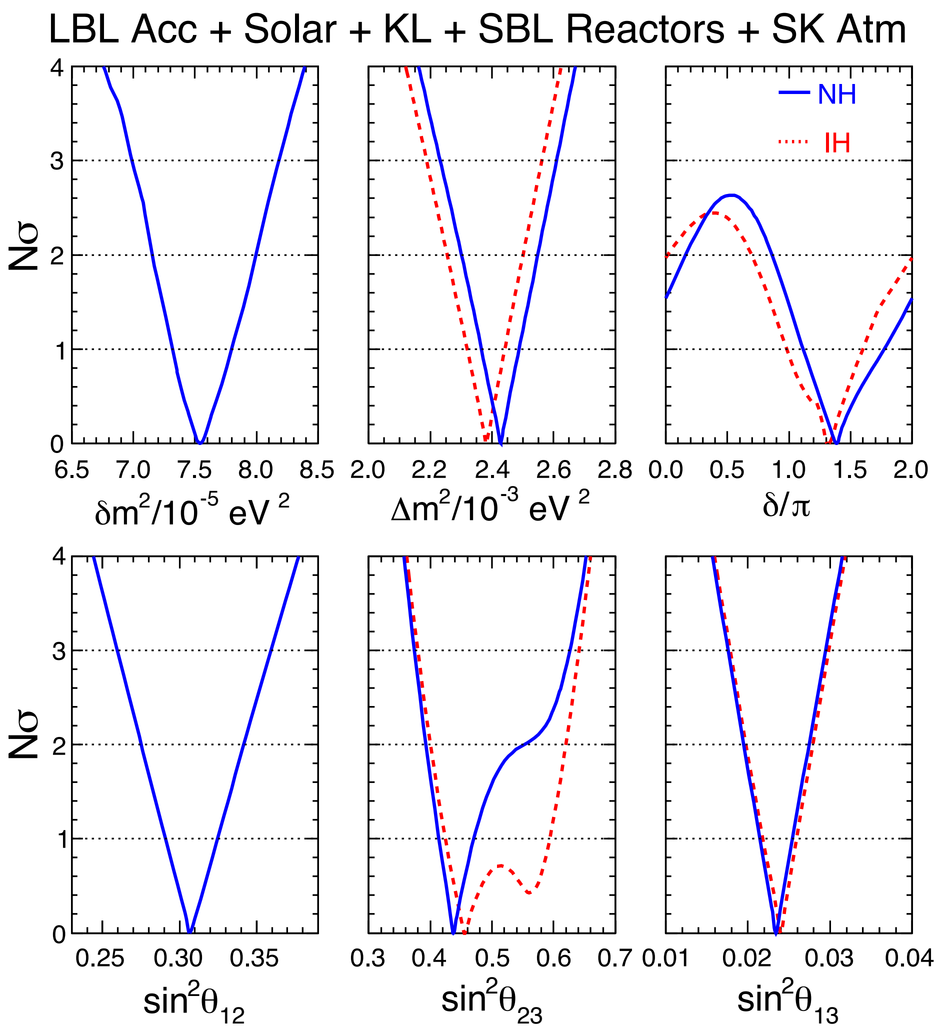

Figure 2. Combined 3ν analysis in terms of $N\sigma$. Blue (red) line refers to Normal (Inverted) Hierarchy. See [3] for details.

There are many evidences for neutrino flavor conversion. We cannot review here all the phenomenology, addressing the interested reader to specific review (see, e.g. [1]). We have inverted the full data set to find the allowed ranges for all the parameters listed above. This analysis is an upgraded version of a previous analysis performed in 2012 [2] including the new data coming from reactor and accelerator experiments using the latest 2014 data.

The $N\sigma \equiv \sqrt{\chi^2 - \chi^2_{\rm min}}$'s for each variable (being the other variables marginalized away) are shown in Fig. 2 (see [3] for further details). The main differences respect to previous analysis are a reduction in the θ_{13} uncertainties and some changes in $(\Delta m^2, \theta_{23})$ ranges. In particular, an overall preference for the first θ_{23} octant and a non-zero CP violation ($\delta \sim 3\pi/2$) emerge from the data. Unfortunately there is not a significant difference between the two hierarchies. Further studies and refined experimental data will be necessary to establish the true mass hierarchy and the θ_{23} octant as well the CP-violation phase.

REFERENCES

1. K. Nakamura and S.T. Petcov, “Neutrino mass, mixing, and oscillations,” in K. A. Olive *et al.* [Particle Data Group Collaboration], Chin. Phys. C **38**, 090001 (2014).
2. G. L. Fogli, E. Lisi, A. Marrone, D. Montanino, A. Palazzo and A. M. Rotunno, Phys. Rev. D **86**, 013012 (2012).
3. F. Capozzi, G. L. Fogli, E. Lisi, A. Marrone, D. Montanino and A. Palazzo, Phys. Rev. D **89**, 093018 (2014).