Halo effective theory for $\alpha-\alpha$ interaction to next-to-next-to-leading order

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Nuclear reactions involving α particles constitute a central issue of nuclear physics: besides governing α-decay processes, they are crucial for the stellar nucleosynthesis. The theory of Big-Bang nucleosynthesis explains the formation of light nuclei like ^{4}He, while heavier nuclei are produced inside the stars. This is due to the absence of stable $A = 5$ and 8 isobars. In particular, crucial for the development of life was the formation of ^{12}C, which proceeds through the so-called triple α process. The latter in turn heavily relies on the existence of the "Hoyle" state [1], an excited state of ^{12}C, predicted by Hoyle before its experimental discovery, with energy very close to the α-^{8}Be threshold. Only recently is the structure of these nuclear systems being understood in terms of the underlying nuclear interaction among nucleons [2]. An alternative and more effective approach consists in treating such nuclei as clusters of α particles. Cluster models have a long history, but the idea of effective theories put them on a firmer theoretical ground. Indeed, effective theories are rooted in the existence of a separation of scales: in this case the separated scales are the typical momenta involved in the reactions of interest and the momenta necessary to excite the α particle. A description in terms of point-like α particles is simply more effective, at sufficiently low energy, than one in terms of interacting nucleons. The "halo" effective theory refers to such a situation, of nuclei composed of a stable core with a halo of orbiting nucleons (or further α particles). The $\alpha - \alpha$ interaction is one essential ingredient of such description, besides the $N-\alpha$ interaction and many-body forces. It is composed of (an infinite tower of) contact (zero-range) interactions, ordered according to the number of momenta (or gradients) involved. The theory is renormalizable order by order in the low-momentum expansion. The strenghts of the interaction vertices, the low-energy constants (LECs), subsume the information of (unknown) short-range dynamics. From this point of view, ab-initio methods (in terms of interactions among point-like nucleons, as given e.g. in chiral effective field theory) could be used to determine the values of these LECs. In turns the LECs of the effective theory of interacting nucleons could be determined by ab-initio calculations in QCD (e.g. on the lattice). In this contribution [3,4] we de-

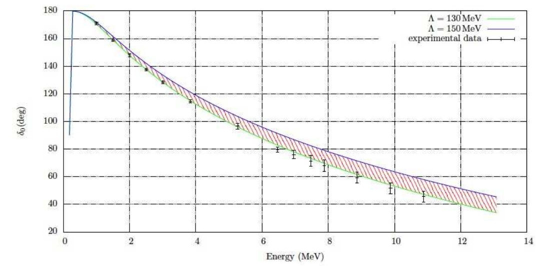

Figure 1. Leading-order predictions for the S-wave phaseshifts as compared to experimental data [5] for different values of the cutoff between $\Lambda = 130$ MeV and $\Lambda = 150$ MeV.

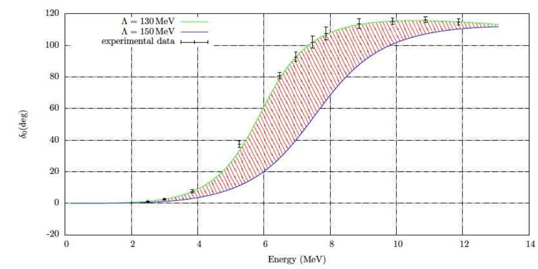

Figure 2. Same as Fig. 1 but for the D-wave phaseshifts.

rive the (strong) $\alpha-\alpha$ interaction to the next-to-next-to-leading order (N2LO) of the low-energy expansion. The strong interaction is considered on the top of the Coulomb interaction. In fact, a very delicate balance exists between the strong attraction and the Coulomb repulsion, which results in the ^{8}Be being only slightly unbound. It constitutes a very narrow (width ~ 7 eV) resonance in the S-wave $\alpha - \alpha$ scattering at an energy of 92 keV in the laboratory frame. Both interactions are regularized with a gaussian cutoff

$f_\Lambda(\mathbf{k^2}) = \exp(-\mathbf{k}^2/2\Lambda^2)$ depending only on momentum transfer $\mathbf{k}$, so as to obtain a local interaction in coordinate space, and the dependence on the cutoff Λ is investigated. By implementing all constraints from the underlying symmetries (rotational, permutation and Galileian relativity) we find that the strong potential depends on a single LEC at leading order (LO), one further LEC at next-to-leading order (NLO) and three LECs at N2LO,

$$V_{\rm eff}^{\rm strong}(\mathbf{k},\mathbf{Q}) = f_\Lambda(\mathbf{k}^2)\left\{\frac{a}{\Lambda^2} + \frac{b}{\Lambda^4}\mathbf{k}^2 + \frac{1}{\Lambda^6}\left[c_1\mathbf{k}^4 + c_2\mathbf{k}^2\mathbf{Q}^2 + c_3(\mathbf{k}\times\mathbf{Q})^{\mathbf{2}}\right]\right\} \quad (1)$$

where $\mathbf{Q}$ denotes the average relative momentum and appropriate powers of the cutoff are inserted to make the LECs adimensional. The LO LEC a is fixed from the position of the resonance at 92 keV for each value of the cutoff. The LO theory is thus completely specified and the phaseshifts can be compared to experimental ones.

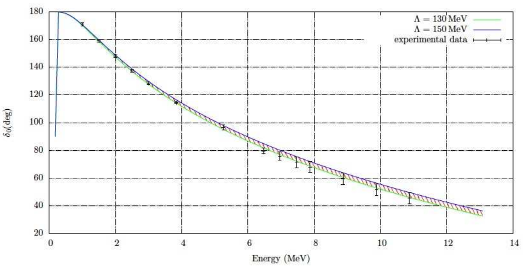

Figure 3. N2LO predictions for the S-wave phaseshifts as compared to experimental data [5] for different values of the cutoff between $\Lambda = 130$ MeV and $\Lambda = 150$ MeV.

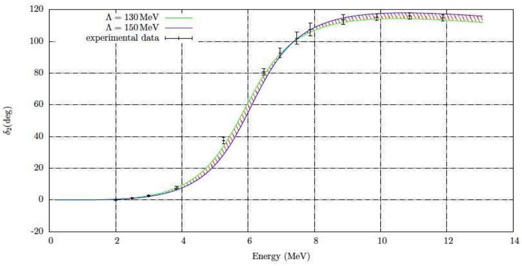

Figure 4. Same as Fig. 3 but for the D-wave phaseshifts.

The results of this LO analysis are shown in Fig. 1

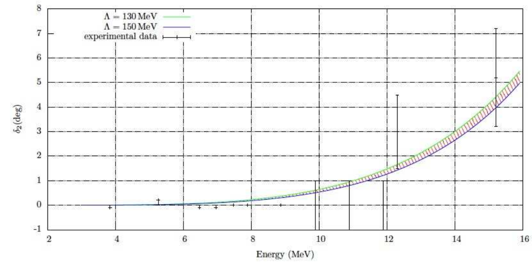

Figure 5. Same as Fig. 3 but for the F-wave phaseshifts.

for the S-wave and in Fig. 2 for the D-wave. For the S-wave a satisfactory description of data up to and beyond 10 MeV is reached for cutoffs between 130 MeV and 150 MeV. This range of cutoff should be compared to the momenta corresponding to the energy of the first excited state of the α particle, $E_{\rm exc} \sim 10$ MeV then $p_{\rm exc} \sim 200$ MeV. It is reasonable to identify the spread in the predictions corresponding to the above range of cutoffs as the theoretical uncertainty inherent in this LO description. Both the S and D waves are well reproduced within the aforementioned theoretical uncertainty, with a much larger uncertainty for the D-wave. We can extend the analysis to NLO and N2LO. In so doing we fix the LECs by reproducing the position of the ^{8}Be resonance and by fitting the low-energy ($E < 5$ MeV) experimental phaseshifts. The associated theoretical uncertainties are much reduced already at NLO. In Figg. 3, 4 and 5 we show the results of the N2LO theory for the S-, D- and F-wave respectively. Out of the three LECs appearing at this order we only fit c_3, whose corresponding operator can be put in relation with the angular momentum squared. The agreement with data is excellent with a remarkably small theoretical uncertainty. As a result, cluster-model calculations can be put on a firm theoretical ground, with a controlled theoretical uncertainty.

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