

# 2014 Annual Report of the String Theory Group

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The research of the group during the 2014 involved a variety of themes related to the gauge/gravity correspondence, ranging from integrability, entanglement entropy of  $AdS_3$  black holes, supergravity, higher spin theories and superconformal  $W_\infty$  algebras. Here we present a brief summary of the published works:

## 1. On the next-to-leading holographic entanglement entropy in $AdS_3/CFT_2$ [1]

We reconsider the one-loop correction to the holographic entanglement entropy in  $AdS_3/CFT_2$  by analysing the contributions due to a bulk higher spin  $s$  current or a scalar field with scaling dimension  $\Delta$ . We consider the two-interval case and work perturbatively in their small cross ratio  $x$ . We provide various results for the entanglement entropy due to the so-called CDW elements of the associated Schottky group. In the higher spin current case, we obtain a closed formula for all the contributions of the form  $\mathcal{O}(x^{2s+p})$  up to  $\mathcal{O}(x^{4s})$ , where 2-CDW elements are relevant. In the scalar field case, we calculate the similar contributions for generic values of  $\Delta$ . The terms up to  $\mathcal{O}(x^{2\Delta+5})$  are compared with an explicit CFT calculation with full agreement. This allows to identify in a clean way the relevant operators that provide the gravity result. The 2-CDW contributions are also analysed and a closed formula for the leading  $\mathcal{O}(x^{4s})$  coefficient is presented.

## 2. On the one-loop curvature function in the $sl(2)$ sector of $N = 4$ SYM [2]

We consider twist  $J$  operators with spin  $S$  in the  $sl(2)$  sector of  $N = 4$  SYM. The small spin expansion of their anomalous dimension defines the so-called slope functions. Much is known about the linear term, but the study of the quadratic correction, the curvature function, started only very recently. At any fixed  $J$ , the curvature function can be extracted at all loops from the  $P\mu$ -system formulation of the Thermodynamical

Bethe Ansatz. Here, we work at the one-loop level and follow a different approach. We present a systematic double expansion of the Bethe Ansatz equations at large  $J$  and small winding number. We succeed in fully resumming this expansion and obtain a closed explicit simple formula for the one-loop curvature function. The result is an explicit series in odd-index  $\zeta$  values. Our approach provides a complete reconciliation between the  $P\mu$ -system predictions and the large  $J$  approach.

## 3. The large $N = 4$ superconformal $W_\infty$ algebra [3]

The most general large  $N = 4$  superconformal  $W_\infty$  algebra, containing in addition to the superconformal algebra one supermultiplet for each integer spin, is analysed in detail. It is found that the  $W_\infty$  algebra is uniquely determined by the levels of the two  $su(2)$  algebras, a conclusion that holds both for the linear and the non-linear case.

## 4. Partition function of free conformal higher spin theory [4]

We compute the canonical partition function  $Z$  of non-interacting conformal higher spin (CHS) theory viewed as a collection of free spin  $s$  CFT's in  $R^d$ . We discuss in detail the 4-dimensional case (where  $s = 1$  is the standard Maxwell vector,  $s = 2$  is the Weyl graviton, etc.), but also present a generalization for all even dimensions  $d$ .  $Z$  may be found by counting the numbers of conformal operators and their descendants (modulo gauge identities and equations of motion) weighted by scaling dimensions. There is also a close relation to massless higher spin partition functions with alternative boundary conditions in  $AdS_{d+1}$ . The same partition function  $Z$  may also be computed from the CHS path integral on a curved  $S^1 \times S^{d-1}$  background. This allows us to determine a simple factorized form of the CHS kinetic operator on this conformally flat background. Summing the individual conformal spin contributions  $Z_s$  over all spins we obtain the total partition function of the CHS theory. We also find the correspond-

ing Casimir energy and show that it vanishes if one uses the same regularization prescription that implies the cancellation of the total conformal anomaly  $a$ -coefficient. This happens to be true in all even dimensions  $d \geq 2$ .

### 5. Higher spins in $AdS_5$ at one loop: vacuum energy, boundary conformal anomalies and AdS/CFT [5]

We consider general-symmetry higher spin fields in  $AdS_5$  and derive expressions for their one-loop corrections to vacuum energy  $E$  and the associated  $4d$  boundary conformal anomaly  $a$ -coefficient. We propose a similar expression for the second conformal anomaly  $c$ -coefficient. We show that all the three quantities  $(E, a, c)$  computed for  $N = 8$  gauged  $5d$  supergravity are  $-1/2$  of the values for  $N = 4$  conformal  $4d$  supergravity and also twice the values for  $N = 4$  Maxwell multiplet. This gives  $5d$  derivation of the fact that the system of  $N = 4$  conformal supergravity and four  $N = 4$  Maxwell multiplets is anomaly free. The values of  $(E, a, c)$  for the states at level  $p$  of Kaluza-Klein tower of  $10d$  type  $IIB$  supergravity compactified on  $S^5$  turn out to be equal to those for  $p$  copies of  $N = 4$  Maxwell multiplets. Under a natural regularization of the sum over  $p$ , the full  $10d$  supergravity contribution is then minus that of the Maxwell multiplet, in agreement with the standard adjoint AdS/CFT duality. We also verify the matching of  $(E, a, c)$  for spin 0 and  $1/2$  boundary theory cases of vectorial AdS/CFT duality. The consistency conditions for vectorial AdS/CFT turn out to be equivalent to the cancellation of anomalies in the closely related  $4d$  conformal higher spin theories. In addition, we study novel example of vectorial AdS/CFT duality when the boundary theory is described by free spin 1 fields and is dual to a particular higher spin theory in  $AdS_5$  containing fields in mixed-symmetry representations. We also discuss its supersymmetric generalizations.

### 6. Vectorial $AdS_5/CFT_4$ duality for spin-one boundary theory [6]

We consider an example of vectorial  $AdS_5/CFT_4$  duality when the boundary theory is described by  $N$  free complex or real Maxwell fields. It is dual to a particular ("type C") higher spin theory in  $AdS_5$  containing fields in special mixed-symmetry representations. We extend the study of this theory in arXiv:1410.3273 by deriving the expression for the large  $N$  limit of the corresponding singlet-sector partition function on  $S^1 \times S^3$ . We find that in both complex  $U(N)$

and real  $O(N)$  invariant cases the form of the one-particle partition function is as required by the AdS/CFT duality. We also demonstrate the matching of the Casimir energy on  $S^3$  by assuming an integer shift in the bulk theory coupling.

### 7. Supergravity one-loop corrections on $AdS_7$ and $AdS_3$ , higher spins and AdS/CFT [7]

As was shown earlier, the one-loop correction in  $10d$  supergravity on  $AdS_5 \times S^5$  corresponds to the contributions to the vacuum energy and  $4d$  boundary conformal anomaly which are minus the values for one  $N = 4$  Maxwell supermultiplet, thus reproducing the subleading term in the  $N^2 - 1$  coefficient in the dual  $SU(N)$  SYM theory. We perform similar one-loop computations in  $11d$  supergravity on  $AdS_7 \times S_4$  and  $10d$  supergravity on  $AdS_3 \times S_3 \times T^4$ . In the  $AdS_7$  case we find that the corrections to the  $6d$  conformal anomaly  $a$ -coefficient and the vacuum energy are again minus the ones for one  $(2, 0)$  tensor multiplet, suggesting that the total  $a$ -anomaly coefficient for the dual  $(2, 0)$  theory is  $4N^3 - 9/4N - 7/4$  and thus vanishes for  $N = 1$ . In the  $AdS_3$  case the one-loop correction to the vacuum energy or  $2d$  central charge turns out to be equal to that of one free  $(4, 4)$  scalar multiplet, i.e. is  $c = 6$ . This reproduces the subleading term in the central charge  $c = 6(Q_1 Q_5 + 1)$  of the dual  $2d$  CFT describing decoupling limit of  $D5 - D1$  system. We also present the expressions for the  $6d$   $a$ -anomaly coefficient and vacuum energy contributions of general-symmetry higher spin field in  $AdS_7$  and consider their application to tests of vectorial AdS/CFT with the boundary conformal  $6d$  theory represented by free scalars, spinors or rank-2 antisymmetric tensors.

### REFERENCES

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