

# On the geometry of Hamiltonian formalism for partial differential equations

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The integrability of Partial Differential Equations (PDEs) is quite often obtained from the existence of distinguished algebraic and/or geometric structures. More particularly, a feature of many (non-linear) integrable differential equations is the existence of an infinite sequence ('hierarchy') of commuting higher (or generalized) symmetries and/or conservation laws. Such sequences are often generated through differential operators.

We recognised that third-order homogeneous operators can be identified with quadratic line complexes, which are algebraic varieties in the Grassmannian of all lines of a projective space [ 2]. The results have been exposed in several universities in 2013 [ 6, 7, 8, 9], in 2014 [ 10, 11, 12, 13] and in 2015 [ 14, 15, 16]. The results have been used in a recent paper [ 3].

R. Vitolo developed the program CDE that runs in the symbolic environment REDUCE (which is now free software) [ 17]. This is one of the few publicly available programs which is able to make computations on integrability differential operators. The theory on which the software is based is exposed in a series of papers [ 4, 5].

R. Vitolo now is one of REDUCE developers. In collaboration with A.C. Norman (Trinity College, Cambridge) he wrote an introduction to Reduce programming [ 18], which is now part of the official Reduce documentation, see <http://sourceforge.net/projects/reduce-algebra/>.

## REFERENCES

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