Fundamental domain and cellular decomposition of spherical space forms

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The spherical space forms are obtained as the quotient space of the spheres (of odd dimension) by fixed-point free actions of finite groups; the family of such groups is explicitly known. These are complete Riemannian manifolds of constant positive curvature.

The spherical space forms problem splits into two problems, that of describing the groups which can occur, and that of describing the ways in which a given group can occur. This problem was actually solved by means of the results of several authors in group theory and representation theory. We address the interested reader to the book of Wolf [7] that gives a complete classification of the spherical space forms. In [6] Tomoda and Zvengrowski studied the cohomology ring of the tridimensional spherical space forms. The basic idea is to produce an explicit resolution for the fundamental group π of these spaces. This is indeed a long standing problem in algebraic topology. They present an explicit resolution for the binary tetrahedral group using a computational tool, a system in computational discreet algebra: the GAP (Groups, Algorithms, Programming).

The basic idea of this project, with O. Manzoli Neto, L.L. Femina, and A.P.T. Galves is to obtain an explicit resolution for the spherical space form groups using geometry. Our approach is based on the original idea of Swan in [5]. Let a finite group π act freely on a sphere S^n. Then, in order to obtain a resolution for π it is sufficient to obtain a π-equivariant CW decomposition of S^n. Of course, the main problem in applying this approach is computational, and this is the reason why, after it was successfully exploited for the cyclic groups, it was somehow abandoned. Alternatively, one must find a suitable geometric model that makes the hard calculation possible. This is the approach followed in this project, and its main achievement. In [4], we studied the generalized quaternionic groups, and in [2], we studied the split metacyclic groups. In particular, we followed the clever geometric setting introduced by F. Cohen in [1, Ch. 9]. In that work, Cohen consider the cyclic group, and a sophisticated description of the cellular complex iss obtained using the join decomposition of a sphere in some spheres of lower dimension, i.e., the method used to obtain the decomposition is essentially geometric. We address to that book for all the basic details of the construction.

In [2] and [4], the ideas of Cohen were used, and after some substantial improving of his technique, it was possible to obtain a cellular decomposition of the sphere S^n, equivariant with respect to the actions of the groups studied. Recently we shown that this technique is in fact powerful enough in order to deal with another class of spherical space forms: the binary tetrahedral spherical space forms. In [3] we considered the particular case of P_{24}. Presently, we are working on the other groups, $P_{8\cdot 3^k}$, and P_{120}.

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