Lie symmetries of fractional partial differential equations

Rosario Antonio Leo [1] Gabriele Sicuro [2] P. Tempesta [3]

[1]Dipartimento di Matematica e Fisica “Ennio De Giorgi”, Università del Salento, Via per Arnesano, 73100 – Lecce, Italy,

[2]Centro Brasileiro de Pesquisas Fisicas , Rua Xavier Sigaud 150, 22290-180 Rio de Janeiro RJ, Brazil

[3]Departamento de Física Teórica II, Facultad de Físicas, Ciudad Universitaria, Universidad Complutense, 28040 – Madrid, Spain

The aim of our work is to establish a general approach for the determination of Lie symmetries for fractional partial differential equations (FPDEs) of Riemann-Liouville type.

The study of fractional differential equations (FDEs) has an intriguing history, doing back to XIX century, with the works of Abel, Liouville, Riemann, etc. However, in the last decade, there has been a resurgence of interest in FDEs, since they have been recognized as fundamental not only in pure mathematics, but also for their manifold applications in statistical mechanics, economics, social science, especially in the description of nonlinear phenomena like anomalous diffusion. Consequently, the search for exact solutions for both stationary and evolution FDEs is of great relevance.

The main theorem, proposed in the paper [1], concerning the existence of symmetries for FPDEs in $(1 + 1)$ dimensions, generalizes the very few results known in the literature. In [2,3], the case of equations involving fractional derivatives with respect to one independent variable has been considered. In [4,5], interesting scale invariant solutions of diffusion equations have been constructed. The intrinsic non commutativity of fractional derivatives with respect to different variables, and, in the case of a single variable, with respect to different fractional orders, has represented until now the main problem in the treatment of symmetries of FPDEs.

Our strategy is inspired by the classical Lie theory: the annihilation of the prolonged action of the vector fields generating the symmetry transformations is imposed. This condition leads to a system of determining equations that allow us to deduce the explicit expression for the symmetry generators. The knowledge of the invariants associated with such generators is a sufficient condition to reduce a given fractional partial differential equation into a new one, characterized by a smaller number of independent variables. In the case of fractional ordinary differential equation (FODE), the reduction process leads to another FODE of reduced order.
As an application of the our theory, in the paper [1] we have proposed a symmetry analysis of fractional KdV-Burgers equation.

Finally in the paper [6] we propose a general theoretical approach for the determination of Lie symmetries of fractional order differential equations with an arbitrary number of independent variables.

REFERENCES

1. R. A. Leo, G. Sicuro, P. Tempesta, A theorem on existence of symmetries of fractional PDEs, C.R. Acad. Sc. Paris, Ser I 352 (2014) 219-222.
2. E. Buckwar, Y Luchko, Invariance of a partial differential equation of fractional order under the Lie group of scaling transformations, J. Math. Anal. Appl. 227 (1998) 81-97
3. R. K. Gazizov, A. A. Kasatkin, S. Y. Lukashchuk, Symmetry properties of fractional diffusion equation, Phys. Scr. T136 (2009) 014016
4. R. Gorenflo, Yu. Luchko, F. Mainardi, Wright functions as scale-invariant solutions of the diffusion-wave equation, J. Comput. Appl. Math. 11 (2000) 175-191
5. Yu. Luchko, R. Gorenflo, Scale-invariant solutions of a partial differential equation of fractional order, Fract. Calc. Appl. Anal. 1 (1998) 63-78
6. R. A. Leo, G. Sicuro, P. Tempesta, A general theory of Lie simmetries for fractional differential equations, arXiv: 1405.1017 [math.AP] (2014)