# On a class of elliptic operators with unbounded diffusion coefficients

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In [1], we extended the results proved by Fornaro and Lorenzi for the operator $a(x)\Delta$ to the case of variable coefficients $a(x)\sum_{ij} a_{ij}(x)D_{ij}$, assuming that the diffusion coefficients admit a limit at infinity. Consider for example the operator

$$L = (1+|x|^2)^{\frac{\alpha}{2}} \sum_{i,j=1}^{N} a_{ij}(x)D_{ij} \tag{1}$$

in $L^p(\mathbb{R}^N)$ with respect to the Lebesgue measure. Fornaro and Lorenzi proved that, for $-\infty < \alpha \leq 2$ and $a_{ij} = \delta_{ij}$, it generates an analytic semigroup in $L^p(\mathbb{R}^N)$ for every $1 < p < \infty$. The case $\alpha > 2$ has been investigated by G. Metafune, C. Spina, C. Tacelli who proved generation results and kernel estimates also for operators containing drift terms and allowing degeneration near the origin.
The natural question whether similar results remain true when the Laplacian is replaced by a more general second order uniformly elliptic operator arised. The answer whas not obvious even for $\alpha \leq 2$ since the techniques previously used are based upon the homogeneous Calderon-Zygmund estimates for the Laplacian which only recently have been extended to the case where the coefficients have a limit at infinity. We proved domain characterization and generation results for the operator $L$ above, under the requirement of existence of limit at infinity of the coefficients $a_{ij}$. A-priori estimates have been proved using the classical ones and covering arguments even for complex-valued coefficients and have been used, trhough Agmon's technique, to prove resolvent estimates for complex $\lambda$.
We proved also a-priori estimates for more general operators of the form

$$L = w_1(y) \sum_{i,j=1}^{K} a_{ij}(x)D_{ij} + w_2(z) \sum_{r,s=K+1}^{N} a_{rs}(x)D_{rs}$$

where $0 < w_i \in C^1(\mathbb{R}^N)$ satisfy

$$|\nabla_z w_1(z)| \leq c_1 w_1^{1/2}(z),$$

$$|\nabla_y w_2(y)| \leq c_1 w_2^{1/2}(y)$$

for some $c_1, c_2 > 0$ under the following assumptions on the coefficients $a_{i,j} \in BUC(\mathbb{R}^N)$, the space of all uniformly continuous and bounded functions over $\mathbb{R}^N$: $a_{is} = a_{rj} = 0$ for $1 \leq i,j \leq K < K+1 \leq r,s \leq N$,

$$\left|\sum_{ij} a_{ij}(x)\xi_i\xi_j\right| \geq \nu|\xi|^2, \quad x,\xi \in \mathbb{R}^N, \nu > 0$$

$$\lim_{|x|\to\infty}(a_{ij}) = (l_{ij}).$$

Note that both $w_1, w_2$ have at most a quadratical growth.

## REFERENCES

1. G. Metafune, C. Spina, C. Tacelli: On a class of elliptic operators with unbounded diffusion coefficients, Evol. Equ. Control Theory 3 (2014), 671680.