# R torsion and analytic torsion for spaces with conical singularities

M. Spreafico [1] [2]

[1]Dipartimento di Fisica, Università del Salento, Italy

[2]ICMC Universidade de São Paulo, Brasil

Let $W$ a compact connected orientable manifold of finite dimension, with possible boundary $\partial W$. Let $g$ a Riemannian structure on $W$. The R torsion $\tau((W,g);\rho)$ is a geometric invariant of the pair $(W,g)$, defined by the graded product of the determinant of the changes of basis in the vector spaces of real chains $\mathsf{C}_\bullet(W;\mathbb{Z}\pi_1(W)_\rho)$, defined for any orthogonal representation of the fundamental group of $W$, $\rho:\pi_1(W)\to O(k)$. The analytic counter part of R torsion, the analytic torsion $T((W,g);\rho)$, was introduced by Ray and Singer in the seventies [1] and is defined by the graded sum of the zeta regularized determinant of the Laplace Beltrami operator on smooth forms over $(W,g)$. The equivalence of the two torsions for a manifold without boundary conjectured by Ray and Singer was proved independently by J. Cheeger [2] and W. Müller [3]. Since then, several extensions and generalisations were studied. In particular, the problem of determining the suitable extension of torsion for manifolds with singularities was addressed by early works of Dar [4] [5] and after that somehow forgot, due to some inconsistency in the last works and lack of suitable technics to tackle the problems due to the presence of the singularities. The first case to start with is the one of conical singularities. From the analytic point of view, the correct framework for the definition of the analytic torsion is that of square integrable forms, developed in early works of Cheeger [6]. In this setting it is possible to defined the suitable extension of the notion of analytic torsion for a space with conical singularities. The first natural question is to understand relationship with the torsion of the section. This objective was achieved by works of L. Hartmann and the author [7] [8] [9] [10], and works of W. Müller and B. Vertmann [11] [12] using a technique introduced by of the author for the zeta regularisation of a class of double sequences [13] [14] [15]. The second natural question is to find the suitable extension of the mentioned Cheeger Müller theorem. The candidate for R torsion is intersection torsion, namely an invariant defined in the setting of intersection homology theory of Gregory and Mac Pherson [16] [17] [18] rather than in the usual homology theory. However, some mayor problems in the correct determination of the necessary theory seems to prevent the natural suggested path. We are now working on possible solutions on these obstructions. The case of odd dimensional manifolds was in fact satisfactorily answered in recent preprint [19].

## REFERENCES

1. D.B. Ray and I.M. Singer, *R-torsion and the Laplacian on Riemannian manifolds*, Adv. Math. 7 (1971) 145-210.
2. J. Cheeger, *Analytic torsion and the heat equation*, Ann. Math. 109 (1979) 259-322.
3. W. Müller, *Analytic torsion and R-torsion of Riemannian manifolds*, Adv. Math. 28 (1978) 233-305.
4. A. Dar, *Intersection R-torsion and the analytic torsion for pseudomanifolds*, Math. Z. 154 (1987) 155-210.

5. A. Dar, *Intersection Whitehead torsion and the s-Cobordism Theorem for Pseudo-manifolds*, Math. Z. 199 (1988) 171-179.
6. J. Cheeger, *Spectral geometry of singular Riemannian spaces*, J. Diff. Geom. 18 (1983) 575-657.
7. L. Hartmann, T. de Melo and M. Spreafico, *The Analytic Torsion of a Disc*, Ann. Global Anal. Geom, 42 (2012) 29-59.
8. L. Hartmann and M. Spreafico, *The analytic torsion of a cone over a sphere*, J. Math. Pure Ap. 93 (2010) 408-435.
9. L. Hartmann and M. Spreafico, *The Analytic Torsion of the Cone over an Odd Dimensional Manifold*, J. Geom. Phys. 61 (2011) 624-657.
10. L. Hartmann and M. Spreafico, *R torsion and analytic torsion of a conical frustum*, J. Gökova Geom. Topology 6 (2012) 28-57.
11. B. Vertman, *Analytic Torsion of a Bounded Generalized Cone*, Comm. Math. Phys. 290 (2009) 813-860.
12. W. Müller and B. Vertmann, *The Metric Anomaly of Analytic Torsion on Manifolds with Conical Singularities* (2011) arXiv:1004.2067v3, to appear in Comm. PDE.
13. M. Spreafico, *Zeta invariants for sequences of spectral type, special functions and the Lerch formula*, Proc. Roy. Soc. Edinburgh 136A (2006) 863-887.
14. M. Spreafico, *Zeta invariants for Dirichlet series*, Pacific. J. Math. 224 (2006) 180-199.
15. M. Spreafico, *Zeta invariants for double sequences of spectral type*, Proc. Amer. Math. Soc. 140 (2012) 1881-1896.
16. M. Goresky and R. MacPherson, *Intersection homology theory*, Topology 19 (1980) 135-162.
17. M. Goresky and R. MacPherson, *Morse theory and intersection homology theory*, Astérisque 101-102 (1983) 135-192.
18. M. Goresky and R. MacPherson, *Intersection homology II*, Invent. Math. 72 (1983) 77-129.
19. L. Hartmann and M. Spreafico, *The Cheer Müller theorem for a cone*, preprint 2014.