Social Context Congestion Games

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The widespread of decentralized and autonomous computational systems, such as highly distributed networks, has rapidly increased the interest of computer scientists for existence and efficiency of equilibria solutions in presence of selfish non-cooperative users. Nevertheless, there are scenarios of practical application (i.e., social networks) in which it can be observed a certain degree of cooperation among users who are related by some kind of knowledge relationships. In such environments, in fact, it may be the case that the happiness of a player does not depend only on her experienced utility, but it is rather somehow related to the one of her "friends". As a consequence, considerable research effort is being devoted to the determination and investigation of suitable frameworks able to combine in a realistic way game theoretical concepts with social network aspects.

Such a task is usually accomplished by coupling a standard non-cooperative game with a social graph expressing some kind of relationship among the players involved in the game. Social graphs, termed as *social knowledge graphs*, were first used in [2–4,6] in order to model the lack of complete information among players. In particular, each player in the game is represented as a node in the graph and it is assumed that there exists an edge between node v_i and node v_j if and only if player i knows player j's adopted strategy. Another model, exploiting social graphs in a different and perhaps more powerful way, is that of *social context games* introduced in [1]. These kinds of games constitute an interesting extension able to capture important concepts related to social aspects in non-cooperative games, like, for instance, collaboration, coordination and collusion among subsets of players. Consider a strategic game SG defined by a given set of players, strategies and payoff functions which we assume, without loss of generality, to be costs for the players and which are called *immediate costs*. Given a social graph G, termed here as *social context graph*, the neighborhood of node v_i in G defines the set of players interacting with player i. The particular type of interaction is then characterized by an aggregation function f which maps tuples of real values into real values. A social context game, defined by the triple (SG, G, f), is a game in which players play as in SG and the cost experienced by player i, called *perceived cost*, is obtained as a function of the *social context* (G, f), that is, by applying f to the tuple yielded by the immediate cost of i and of all the players interacting with her according to G.

Clearly, depending on which underlying game SG, which social context graph G and which aggregation function f are used, several interesting social context games can be defined. In this paper, we focus on the cases in which SG belongs to one of the following subclasses of congestion games: linear congestion games, Shapley cost sharing games and multicast games, while f may be one of the following aggregation functions: minimum, maximum and sum (or average).

We consider social context games in which SG can be either a linear congestion game (LCG) or a Shapley cost sharing game (SCG) and f is one among the minimum, maximum and sum functions. In all cases we completely characterize which topologies of G can always guarantee existence of pure Nash equilibria as follows. For any G, if SG is a linear congestion game and f is the sum function, then (SG, G, f) is an exact potential game (this extends Theorem 6 in [5], which holds for network linear congestion games in which G is a partition into cliques). If G is either the complete or the empty graph, then (SG, G, f) is an exact potential game for any SG and f. In all the other cases, for any fixed G, there always exists a pair (SG, f) such that (SG, G, f) does not admit pure Nash equilibria.

We then bound the price of anarchy for all the six arising social context games under four natural social cost functions, namely, the sum of the players' immediate costs (SUM-IMM), the maximum of the players' immediate costs (MAX-IMM), the sum of the players' perceived costs (SUM-PER) and the maximum of the players' perceived costs (MAX-PER). For coherence with our existence results, we restrict our analysis to the cases in which pure Nash equilibria always exist. This means that we consider any possible social context graph when f is the sum function and SG is a linear congestion game, while we restrict to either empty and complete social context graphs in all the other cases. Note that any social context game (SG, G, f) in which G is the empty graph collapses to game SG, since perceived and immediate costs coincide. Thus, because of the fact that the price of anarchy of either linear congestion games and Shapley cost sharing games has been already characterized under the social functions SUM-IMM and MAX-IMM, we further restrict the analysis of these cases to games in which G is the complete graph.

	SUM-IMM		MAX-IMM		SUM-PER		MAX-PER	
f	LCG	SCG	LCG	SCG	LCG	SCG	LCG	SCG
MIN	$\Theta(nm)$	$\Theta(m)$	$\Theta(nm)$	$\Theta(nm)$	n	n	n	n
MAX	$\Theta(nm)$	m	$\Theta(nm)$	$\Theta(nm)$	$\Theta(nm)$	$\Theta(nm)$	$\Theta(nm)$	$\Theta(nm)$
SUM	$5 \div \frac{17}{3}$	m	$\Theta(\sqrt{n})$	$\Theta(nm)$	$\Omega(\sqrt{n}) \div O(n)$	m	$\Omega(\sqrt{n}) \div O(n)$	m

Figure 1. Bounds on the price of anarchy of the social context games considered in this paper.

Our findings are summarized in Figure 1, where n is the number of players and m is the number of resources in the game. It can be seen that all the results are tight or asymptotically tight except for the two cases in which SG is a linear congestion game, f is the sum function and the social cost function is either SUM-PER or MAX-PER. Moreover, all the bounds are significatively high, except for the case in which SG is a linear congestion game, f is the sum function and the social cost function is SUM-IMM for which a constant price of anarchy holds.

The reasons for this generalized bad worst-case performance of pure Nash equilibria can be found in the following arguments. Recall that, for f being either the min or the max function, we are considering the case in which G is the complete graph which yields the same perceived cost for all the players. When f is the min function, the unique perceived cost is low as long as there is one player who is paying a low immediate cost even when the remaining $n-1$ ones are paying the highest possible immediate cost. In such a situation, it is then possible to boost the price of anarchy to the highest possible values when the social cost functions are defined on the basis on the immediate costs. For social functions defined on the basis of perceived costs, a high price of anarchy can be realized when the minimum immediate cost is yielded by the immediate cost of a single player who is adopting a non-optimal strategy and the remaining $n-1$ players are making choices which cause them a high immediate cost and, at the same time, prevent the minimum cost player to migrate on her optimal strategy. When f is the max function, by partitioning the players into two sets so that, when all players make a non-optimal choice, they all pay the highest possible immediate cost, and any optimal strategy for each player in one partition overlaps with the non-optimal strategy for the players in the other partition, it is possible to boost the price of anarchy to the highest possible values for all the considered social cost functions. All these extreme situations are avoided when f is the sum function. For the case in which SG is a linear congestion game, the facts that all possible social context graphs have to be considered and that significant better bounds are possible make the situation much more intriguing and worthy of further investigation.

Finally, we (partially) extend the above results to the case in which SG is a multicast game, an interesting restriction to networks of Shapley cost sharing games. In particular, we show that any Shapley cost sharing game admitting no pure Nash equilibria can be turned into a multicast game admitting no potential functions (i.e., admitting an infinite sequence of improving deviations). We also show that although multicast games are a restriction of the Shapley cost sharing ones, their prices of anarchy are asymptotically related. More precisely, we show instances of multicast games whose prices of anarchy asymptotically match the upper bounds on the price of anarchy of Shapley cost sharing games in all the cases under analysis.

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