On Lookahead Equilibria in Congestion Games

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The definition of the process of interaction among self-interested entities is dependent on the context, and in particular on the set of information available to the players. When they have very little knowledge about each others' costs and strategies, one of the most natural and studied dynamics are *sequential best-responses*, where players play sequentially and each player selects a strategy which is a best-response to the current strategy of the others. In such dynamics, the assumption is that each player has no memory about the past and no knowledge about the available strategies and costs of other players and, thus, myopically responds to the current state, without making any prediction about the consequences of the subsequent responses of the remaining players. One of the basic objective of study of game theory is the concept of equilibrium. An equilibrium can be viewed as a steady state of a dynamics, where no agent has an incentive to unilaterally deviate from. The steady state of a best-response dynamics is known as *pure Nash equilibrium*. It is well known that best-response dynamics do not always lead to a pure Nash equilibrium and that the class of congestion games [2] is a large class of games guaranteeing convergence under best-responses.

In our work, we focus on the settings in which each player has full knowledge of the strategies and costs of the other players, so that, based on such a knowledge, she can make predictions about the others' reactions to her move. We also assume that each player is an entity with limited computational abilities, thus she has the ability of making predictions only on the consequences of a fixed constant number of subsequent consecutive moves. In particular, we study the *k-lookahead dynamics* in which the players sequentially perform *k-lookahead best-responses*. When $k = 1$, the k-lookahead best-response coincides with the best-response. In general, for $k > 1$, the current moving player p evaluates all the possible outcomes resulting from $k - 1$ subsequent moves, by taking into account all the possible orders in which players move and all of their possible strategies. We say that player p has a long-sightedness of k and she makes a prediction by assuming that any player moving $j < k$ steps after her has a long-sightedness of $k - j$. Thus, player p can compute her best move by backward induction starting from the players having long-sightedness of 1, and proceeding backward up to k. When predicting the strategy chosen by any player q having long sightedness $k - j$, it is necessary to make some assumption on which is the next moving player. We take into account two different models: the worst-case and the average-case ones. In the *worst-case model*, player p assumes that the next move after q is performed by a player providing player q the worst possible cost in the final outcome. In the *average-case model*, player p assumes that the next move after q is taken by a player selected uniformly at random. For each of these models, we finally distinguish between the cases of *consecutive* and *non-consecutive moves*, depending on whether player p assumes that the next move after q may be performed by q itself or not.

In our work, we investigate the existence of k-lookahead equilibria and the price of anarchy of 2-lookahead equilibria in congestion games with linear latencies [2]. Congestion games model the settings in which a set of players compete for the usage of a set of common resources. We choose congestion games as representative of a large set of well studied games for which the existence of pure Nash equilibria is always guaranteed.

We initially focus our attention to the existence of k-lookahead equilibria in strategic games. We are able to show that, in the worst-case model with consecutive moves, for a strategic game, any pure Nash equilibrium is also a k-lookahead equilibrium. This result implicitly shows that the k-lookahead best-responses do not guarantee better performance at equilibrium compared to those achieved by the simple best-responses. We then consider the existence of 2-lookahead equilibria in singleton congestion games. We show that in the worst-case model without consecutive moves, any symmetric singleton game always admits 2-lookahead equilibria. For the average-case model, instead, we show that symmetric singleton congestion games do not always admit a 2-lookahead equilibrium regardless of whether consecutive moves are allowed or not.

We also obtain bounds on the price of anarchy for the 2-lookahead equilibria of linear congestion games, both in the worst-case and in the average-case model. We first show that, in the worst-case model, for any linear congestion game, the price of anarchy is at most 8. For the average-case model, we obtain smaller bounds. In particular, we show that, for any linear congestion game with n players, the price of anarchy is at most $4 + \min\left\{2, \frac{5}{4n-7}\right\}$. This result significantly improves the previous upper bound of $(1+\sqrt{5})^2 \approx 10.47$ given in [1]. All the bounds mentioned hold either with or without consecutive moves. We also show that, when restricting to singleton strategies, the price of anarchy drops to at most 4 in the worst-case model with or without consecutive moves.

REFERENCES

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