One-Dimensional Degenerate Diffusion Operators

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The aim of the paper [1] is to present some results about generation, sectoriality and gradient estimates for the resolvent of suitable realizations of the operators

$$A^{\gamma,b}u(x) = \gamma x u(x) + bu(x), \tag{1}$$

with constants $\gamma > 0$ and $b \geq 0$, in the space $C([0,\infty])$.

To this end, we review some results established mainly in [4,8,6,7], and we propose them in a unified way. Precisely, we start considering the kernel $p^{\gamma,b}(x,y,t)$, given, e.g., in [6,4], of the solution operator $P_t^{\gamma,b}$ for the equation $\partial_t - A^{\gamma,b}$. After proving some estimates for $p^{\gamma,b}(x,y,t)$, we show that $(P_t^{\gamma,b})_{t\geq 0}$ is a C_0-semigroup in $C([0,\infty])$ and that its infinitesimal generator is $A^{\gamma,b}$ endowed with the domain

$$\begin{aligned} D(A^{\gamma,0}) &= \{u \in C([0,\infty]) \cap C^2(]0,\infty[) \mid \lim_{x\to 0^+} A^{\gamma,0}u(x) = 0, \\ &\quad \lim_{x\to+\infty} A^{\gamma,0}u(x) = 0\}, \qquad \text{if } b = 0, \\ D(A^{\gamma,b}) &= \{u \in C^1([0,\infty[) \cap C^2(]0,\infty[) \cap C([0,\infty]) \mid \\ &\quad \lim_{x\to 0^+} xu''(x) = 0, \ \lim_{x\to+\infty} A^{\gamma,b}u(x) = 0\}, \qquad \text{if } b > 0. \end{aligned}$$

Moreover, we prove in [1] that the space of C^2-functions on $[0,\infty[$ that are constant in a neighbourhood of ∞ is a core for $(P_t^{\gamma,b})_{t\geq 0}$.

At this point, the analyticity of $(P_t^{\gamma,b})_{t\geq 0}$ in $C([0,\infty])$ follows immediately from the results in [8,5], but with a careful analysis we also prove that, for any $B > 0$ and $\gamma_0 > 0$ fixed, there exists a constant $C = C(B,\gamma_0) > 0$ such that for every $b \in [0,B]$ and $\gamma \geq \gamma_0$,

$$||tA^{\gamma,b}P_t^{\gamma,b}|| \leq C(B,\gamma_0), \quad t \geq 0,$$

that is, the analyticity constant is uniform in bounded intervals $[0,B]$ and in half-lines $[\gamma_0,\infty[$ with $\gamma_0 > 0$.

We also get pointwise gradient estimates both for the semigroup $(P_t^{\gamma,b})_{t\geq 0}$ and for the resolvent $R(\lambda, A^{\gamma,b})$ and, in the case $b > 0$, we prove that $\partial_x R(\lambda, A^{\gamma,b})$ is a continuous operator from $C([0,\infty])$ into itself and give an estimate of the operatorial norm.

The results presented in this paper play an important role in [2,3] to show the analyticity in spaces of continuous functions of the semigroup generated by some degenerate diffusion operators defined on domains of $\mathbb{R}^d$ with corners.

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