Three-nucleon interaction

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Despite long-lasting efforts in the determination of a realistic three-nucleon force, none of the presently available models leads to a satisfactory description of bound and scattering states of the $A = 3$ system [1]. The traditional nuclear physics approach, aimed at the phenomenological modeling of the interaction to experimental data seems unpractical in this case, due to the large number of operatorial spin-isospin structures. It can be argued that the effective theory approach could provide valuable new insight into the problem of the three-nucleon force (TNF), since in this framework there is a well defined perturbative expansion scheme valid at low energy, which allows to identify which operators enter at each order of the expansion. In the conventional terminology, the TNF starts to contribute at the N2LO [(next-to)^2-leading-order] level, thus justifying the traditional view of the nuclear interaction as primarily consisting of two-body interactions. At this order the TNF depends on two unknown coupling constants, so called low-energy constants (LECs), and in this form it has been implemented in many ab-inition nuclear physics calculations. Presently, the TNF has been worked out up to the N3LO [2]. It turns out that no new LECs appear at this level, so that the effective theory yields predictions indeed. However, the relevant (rather lengthy) expressions have never been implemented in ab-initio calculations. It would be a marvelous accomplishment of the effective theory approach if this form of the TNF would constitute a "realistic" model, in the sense that it would provide, associated to a realistic two-nucleon potential, a satisfactory fit to three-nucleon observables. In Ref. [3] we have proposed a more pragmatic point of view. We noticed that the main discrepancies between theory and experiment in the three-nucleon system concern very low energies, and disappear as the energy increases. The most prominent of these discrepancies is the so-called A_y puzzle, which exhibits precisely this pattern. At such low energies, the nuclear interactions are effectively point-like. We therefore proposed to refine the "contact" TNF. Indeed, at very low energies, even the pions can be integrated out of the theory, giving rise to the "pionless" effective theory. The inclusion of contact interactions, which are unconstrained by chiral symmetry, could also provide the necessary flexibility to describe the $3N$ interaction. In this respect it is fitting to recall that all adopted $3N$ interaction models only contain a few free parameters, contrary to what happens for the $2N$ interaction, which is parametrized by more than 20 adjustable parameters. For example, the chiral $3N$ force at N2LO only contains 2 adjustable LECs, and its extension to N3LO does not involve any new LEC [2].

In Ref. [3] we have classified all subleading three-nucleon contact operators, which involve two powers of nucleon momenta. These terms would show up at the N4LO of the low-energy expansion, but they represent the first correction to the TNF in the pionless version of the effective theory. It could be that, at very low-energy, the latter effective theory, and the counting that it entails, is more appropriate than the pionful version. The resulting three-nucleon potential, which depends on 10 LECs ($E_{1,\ldots,10}$) whose values should be fitted to experimental data, can be cast in a local form in coordinate space,

$$
\begin{aligned}
V = \sum_{i\neq j\neq k} \quad & (E_1 + E_2 \boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j + E_3 \boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j + E_4 \boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j \boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)\left[Z_0''(r_{ij}) + 2\frac{Z_0'(r_{ij})}{r_{ij}}\right] Z_0(r_{ik}) \\
& + (E_5 + E_6 \boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j) S_{ij}\left[Z_0''(r_{ij}) - \frac{Z_0'(r_{ij})}{r_{ij}}\right] Z_0(r_{ik}) \\
& + (E_7 + E_8 \boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_k)(\mathbf{L}\cdot\boldsymbol{S})_{ij}\frac{Z_0'(r_{ij})}{r_{ij}} Z_0(r_{ik}) \\
& + (E_9 + E_{10}\boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_k)\boldsymbol{\sigma}_j\cdot\hat{\mathbf{r}}_{ij}\boldsymbol{\sigma}_k\cdot\hat{\mathbf{r}}_{ik} Z_0'(r_{ij}) Z_0'(r_{ik})
\end{aligned}
\tag{1}
$$

where S_{ij} and $(\mathbf{L}\cdot\boldsymbol{S})_{ij}$ are respectively the tensor and spin-orbit operators for particles i and j. As it is apparent, a choice of basis has been made such that most terms in the potential can be viewed as an ordinary interaction of particles ij with a further dependence on the coordinate of the third particle. In particular, the terms proportional to E_7 and E_8 are of spin-orbit type, and, as already suggested in the literature [4], have the right properties to solve the A_y puzzle.

By matching the (parameter-free) expression of the TNF at N3LO to the pionless theory, we can get the pion-induced contributions to the LECs that we have introduced, as functions of the lowest order LECs [5],

$$\begin{aligned}
E_1 &= \frac{755 g_A^6}{24576\pi F_\pi^6 M_\pi} + \frac{g_A^4}{256\pi F_\pi^6 M_\pi} - \frac{g_A^4 C_T}{64\pi F_\pi^4 M_\pi} - \frac{g_A^2 C_T}{8m F_\pi^2 M_\pi^2} \sim 0.10, && (2)\\
E_2 &= \frac{601 g_A^6}{36864\pi F_\pi^6 M_\pi} + \frac{23 g_A^4 C_T}{384\pi F_\pi^4 M_\pi} - \frac{5 g_A^2 C_T}{192\pi F_\pi^4 M_\pi} - \frac{g_A^2(5C_T + 2C_S)}{48m F_\pi^2 M_\pi^2} \sim 0.06, && (3)\\
E_3 &= -\frac{3 g_A^6}{2048\pi F_\pi^6 M_\pi} + \frac{3 g_A^4 C_T}{64\pi F_\pi^4 M_\pi} + \frac{9 g_A^2 C_T}{16m F_\pi^2 M_\pi^2} \sim 0.00, && (4)\\
E_4 &= -\frac{g_A^6}{1024\pi F_\pi^6 M_\pi} - \frac{3 g_A^2 C_T}{16m F_\pi^2 M_\pi^2} \sim 0.00, && (5)\\
E_5 &= \frac{79 g_A^6}{12288\pi F_\pi^6 M_\pi} + \frac{g_A^4}{256\pi F_\pi^6 M_\pi} - \frac{g_A^4 C_T}{64\pi F_\pi^4 M_\pi} - \frac{g_A^2 C_T}{8m F_\pi^2 M_\pi^2} \sim 0.02, && (6)\\
E_6 &= \frac{319 g_A^6}{36864\pi F_\pi^6 M_\pi} + \frac{g_A^4}{256\pi F_\pi^6 M_\pi} - \frac{g_A^2(C_S - 2C_T)}{24m\pi F_\pi^2 M_\pi^2} \sim 0.04, && (7)\\
E_7 &= -\frac{83 g_A^6}{6144\pi F_\pi^6 M_\pi} - \frac{3 g_A^4}{128\pi F_\pi^6 M_\pi} + \frac{3 g_A^2 C_T}{4m F_\pi^2 M_\pi^2} \sim -0.08, && (8)\\
E_8 &= -\frac{7 g_A^6}{3072\pi F_\pi^6 M_\pi} - \frac{g_A^4}{128\pi F_\pi^6 M_\pi} + \frac{g_A^2 C_T}{4m F_\pi^2 M_\pi^2} \sim -0.02, && (9)\\
E_9 &= \frac{193 g_A^6}{4096\pi F_\pi^6 M_\pi} - \frac{3 g_A^2 C_T}{8m F_\pi^2 M_\pi^2} \sim 0.14, && (10)\\
E_{10} &= \frac{c_1 g_A^2}{2F_\pi^4 M_\pi^2} + \frac{g_A D}{8F_\pi^2 M_\pi^2} + \frac{427 g_A^6}{12288\pi F_\pi^6 M_\pi} + \frac{9 g_A^4}{512\pi F_\pi^6 M_\pi} - \frac{g_A^2(C_S + C_T)}{8m F_\pi^2 M_\pi^2} \sim 0.09. && (11)
\end{aligned}$$

The (rough) numerical values are given in units of $F_\pi^4 M_\pi^3$. Notice that only E_{10} receives contributions from both N2LO (~ -0.05) and N3LO (~ 0.15) and there is no sign of convergence. At N4LO there will appear the "genuine" contact contributions: if the convergence pattern is so bad, it is possible that they be phenomenologically relevant. In any case, the choice of a purely contact $3N$ interaction (including the subleading one), with a cutoff $\sim M_\pi$, could turn out to be rather effective from the practical point of view.

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