Correlated nuclear ground state

G. Co' [1 2] and S. De Leo [3]

[1]Dipartimento di Matematica e Fisica "E. De Giorgi", Università del Salento, Italy

[2]Istituto Nazionale di Fisica Nucleare sez. di Lecce, Italy

[3]Dept. of Applied Mathematics, State University of Campinas, Brazil

The theoretical basis of the mean-field approach to the description of the stucture of the atomic nucleus has been deeply studied in the sixites of the past centuries [1]. This approach is based on the Hartree-Fock (HF) theory for the description of the ground state of the system, and on the Random Phase Approximation (RPA) theory for the description of the excited states. Modern applications of this approach are able to describe with high accuracy ground [2] and excited state properties [3,4].

The only input of these mean-field theories is the nucleon-nucleon interaction. The interaction used in these theories is called *effective* since is not the interaction extracted by the study of the two, and eventually, three nucleon systems, which is instead called *bare*. This latter interaction has a strongly repulsive core at short internucleonic distances, and cannot be used in mean-field theories because it would produce divergences.

The use of the bare interactions in nuclear structure calculations requires to go beyond the mean-field theories by considering many-body effects which are named correlations. The effects not explicitly considered in mean-field theories, the correlations, are taken into account by modifying the nucleon-nucleon interaction and inserted effectively in the values of its parameters.

We can classify the correlations in two different cathegories. A first one is related to the short-range physics, and these are the many-body effects which reduce the role of the strongly repulsive core. These type of correlations are well studied in the Correlated Basis Function (CBF) theories, and the application of this theory to finite nuclear systems indicate that, beyond a certain nuclear size, these correlations are very similar for each nucleus [5]. The second type of the correlations are called of long-range and take into account the coupling of the mean-field states with collective vibrations, phonons, of the nucleus.

In our work we propose a description of the nuclear and excited states by considering explicitly long-range correlations. Our perspective is to use the bare nucleon-nucleon interactions corrected by the CBF short-range correlations. In this manner both type of correlations would be considered.

We start from the hypothesis that the nuclear ground state can be described as linear combination of Slater determinants containing one particle-one hole (1p1h) excitations, as well as, two particle-two hole (2p2h) correlations

$$|\Psi_0\rangle = \Big[1 \quad + \quad \sum_{mi} B_{m\,i}\, a^+_m a_i \\ \qquad + \quad \sum_{minj} A_{minj}\, a^+_m a_i a^+_n a_j \Big] |\,\Phi_0\rangle \ . \quad (1)$$

In the above equation, $|\Phi_0\rangle$ indicates the Slater determinant of the mean-field approach, where all the single particle states below the Fermi surface are occupied and those above are empty. The a represent the creation and annihilation operators, and the indexes i, j operate below the Fermi surface and m, n above it.

We apply the Ritz variational principle [6] by using the ansazt (1) for the nuclear ground state, and we obtain a set of integro-differential equations. A first set of equations is obtained by making the variations of the single particle states, and produce expressions similar to those obtained in the traditional HF approach [1,6]. These equations are implemented by those obtained by making the variation on the A coeffiecients. The variation on the B coefficients produce equations independent of B. This means that our approach is compatible with any value of the B, and we set them to zero. The physical interpretation of this result, is that 1p1h pairs describe excited states, while the long-range correlations are produced by the, virtual, coupling of these excited states (phonons), and this is, at first order, described in terms of 2p2h pairs.

We construct the excited states on top of the anzatz (1). We attack the problem by using a time dependent HF approach. We describe the time evolution of the state (1) as

$$\begin{aligned} |\Psi(t)\rangle \quad &= \quad \big[\, 1 + C_{mi}(t)\; a^+_m a_i \\ &\quad + \quad \frac{1}{2} C_{mi}(t) C_{nj}(t)\; a^+_m a_i a^+_n a_j \,\big] \\ &\qquad e^{-\,i\,E_0 t/\hbar}\, |\Psi_0\rangle \ . \qquad (2) \end{aligned}$$

The coefficients C_{mi} are determined by the variational equation

$$\langle \delta\Psi(t)| \left(H - i\hbar\frac{\partial}{\partial t} \right) |\Psi(t)\rangle = 0 \ , \tag{3}$$

where H is the nuclear hamiltonian. We evaluate the above equation, by considering only the linear terms in C, and we obtain the expression

$$\begin{aligned} (\epsilon_m - \epsilon_i)\, C_{mi} \quad &+ \quad \sum_{nj} \left(\mathcal{A}_{minj} C_{nj} \ + \mathcal{B}_{minj} C^*_{nj} \right) \\ &= \quad -i\hbar\frac{\partial}{\partial t} C_{mi} \ , \end{aligned} \tag{4}$$

where we have indicated with ϵ the energy of the single particle level. The $\mathcal{A}$ and $\mathcal{B}$ coefficients contain matrix elements of the interaction between single particle states wheighted by the A_{minj} coeffients of Eq. (1).

We search for a oscillating solution of the Cs

$$C_{mi} = X_{mi} e^{i\omega t} + Y_{mi} e^{-i\omega t} \ . \tag{5}$$

Inserting the expression (5) in (4), separating positive and negative frequencies, and considering only linear terms in the frequency ω, we obtain a set of equations for the X and Y coefficients

$$\sum_{nj} \left(\mathcal{A}_{minj} X^\omega_{nj} + \mathcal{B}_{minj} Y^\omega_{nj} \right) \quad = \quad \hbar\omega X^\omega_{mi} \tag{6}$$

$$\sum_{nj} \left(\mathcal{B}^*_{minj} X^\omega_{nj} + \mathcal{A}_{minj} Y^\omega_{nj} \right) \quad = \quad -\hbar\omega Y^\omega_{mi} \tag{7}$$

The structure of the above equation is analogous to that of the traditional RPA equations, where $\hbar\omega$ is the energy of the excited state, and X^ω_{nj} and Y^ω_{nj} the amplitides of the ph components of the excited state. The novelties of the new ground state are contained in the structure of the $\mathcal{A}$ and $\mathcal{B}$ matrix elements.

The work is in progress. We are now testing the normalization and the closure relation for the excited states, the conservation of the sum rules and we are obtaining the expressions for the transition probabilities.

REFERENCES

1. R. Rowe, Nuclear collective motion, Methuen, London, (1970).
2. G. Co' et al., Phys. Rev. C 85 (2012) 024322.
3. V. De Donno, G. Co', M. Anguiano and A. M. Lallena, Phys. Re. C 83 (2011) 044324.
4. M. Anguiano, G. Co', V. De Donno, and A. M. Lallena, Phys. Re. C 83 (2011) 064306.
5. F. Arias De Saavedra, C. Bisconti, G. Co', A. Fabrocini, Phys. Rep. 450 (2007) 1.
6. A. Messiah, Quantum Mechanics, North Holland, Amsterdam (1961).