Nuclear properties with finite range interactions in HF+BCS model

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In nuclei with open shells, the role played by the pairing correlations is very important. The simplest mean-field approach able to take into account these correlations is the Bardeen–Cooper–Schrieffer (BCS) model [1] in which starting from a single particle (s.p.) basis and using a pure pairing interaction, pairing effects and partial occupation probabilities of the s.p. states can be calculated. A widely used method in this framework is the so-called Hartree-Fock (HF) plus BCS approach, where a HF calculation is carried out to obtain the s.p. basis. While in this approach the production of the s.p. states and their pairing correlations are treated in two different types of calculations, the two processes are unified in the Hartree–Fock–Bogoliubov (HFB) theory [1]. In this approach, effective interactions, i.e. without the strong short-range repulsion, are used. It is well known [2] that a zero-range interaction in the pairing channel, makes the results dependent on the size of the s.p. configuration space considered. Instead a finite-range interaction automatically provides convergence of the results once a certain size of the s.p. configuration space is adopted. There are many examples of HFB calculations performed with finite range interactions, like the Gogny interactions D1 [2], D1S [3] and D1M [4] and the M3Y interaction [5].

However, there are only few works in which finite-range interactions are used in HF+BCS calculations. In our work [6] our intent is to develop the HF+BCS approach combining the advantages of a unique finite-range interaction (D1S and D1M), that offers prediction power, with those of the numerical simplicity. Our main purpose is to investigate the relevance of the pairing correlations in some regions of the nuclear chart far from the stability line and to test the validity of the HF+BCS approach against results of the HFB approach, when they are available.

As example of our results, in Fig. 1 we show, with respect to the mass number A, the particle number fluctuation. This quantity is directly related to the relevance of the pairing effect and is defined as:

$$\langle(\Delta N)^2\rangle \;=\; 4\sum_k (2j_k+1)\,u_k\,v_k\,, \qquad (1)$$

where $|v_k|^2$ is the probability that the state $|k\rangle$ is occupied, and is related to u_k by the condition

$$|u_k|^2 + |v_k|^2 = 1\,. \qquad (2)$$

In Eq. (1) j_k is the angular momentum of the s.p state. The results obtained with our HF+BCS approximation using the Gogny interaction D1M are compared with those obtained with the HFB model [7] using the same interaction for different isotope and isotone chains.

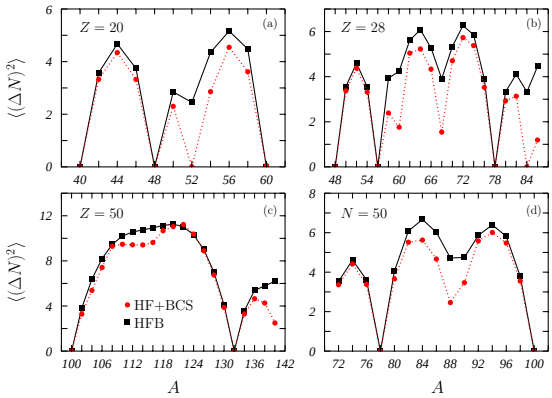

Figure 1. $\langle(\Delta N)^2\rangle$ values for (a) calcium, (b) nickel and (c) tin isotopes and (d) $N = 50$ isotones using HF+BCS (red circles) and HFB (black squares) with the Gogny D1M interaction.

As a first general remark, we observe that in all the chains we have considered, the behavior of the two types of calculations is very similar. The only deviations occur for ^{52}Ca and ^{84}Ni, which do not show any pairing, and for ^{60}Ni and the two heavier isotopes of tin, ^{138}Sn and ^{140}Sn, in which $\langle(\Delta N)^2\rangle$ reduces in HF+BCS while it increases in HFB. In the case of the ^{52}Ca isotope, the 32 neutrons fill completely all the HF s.p. states including the $2p_{3/2}$ level at the energy of -5.44 MeV. The other s.p. levels close to the Fermi surface, the $2p_{1/2}$ and the $1f_{5/2}$ are too separated in

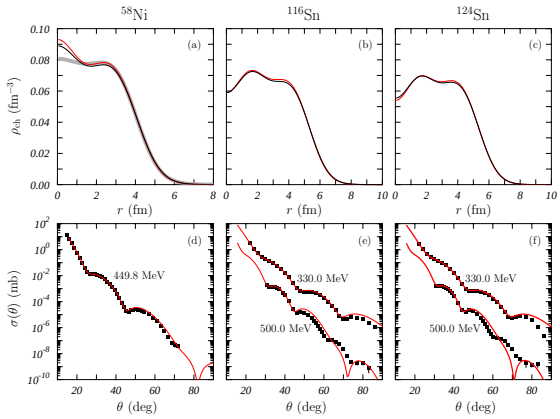

Figure 2. Upper panels: charge densities for ^{58}Ni, ^{116}Sn, and ^{124}Sn nuclei obtained in HF+BCS calculations by using the D1M (red lines) and D1S (black lines) interactions. The thick gray line in panel (a) is the empirical charge density [8]. Lower panels: elastic electron scattering cross sections for the same nuclei calculated with the HF+BCS charge densities with the D1M interaction (red solid lines) and compared with the experimental results (full black squares) from [9–11]. In each panel the incident electron energies are indicated.

energy to permit partial occupations in our BCS calculations. On the contrary, in the HFB approach a certain admixture between these states is obtained. A similar situation is observed in panel (b), for the nucleus ^{84}Ni. This nucleus has 56 neutrons which fill completely all the HF s.p. states up to the $2d_{5/2}$ level. The next two levels, the $3s_{1/2}$ and the $2d_{3/2}$ are very high in energy and our BCS model is not able to generate relevant pairing correlations among these states.

Contrary to what happens in ^{52}Ca, in which the BCS does not change the complete occupation of the neutron $2p_{3/2}$ level, in ^{60}Ni the occupation probability of this s.p. state reduces to 0.89, with a small occupation of the $1f_{5/2}$ state. In the case of the tin isotopes heavier than ^{132}Sn, the $2f_{7/2}$ neutron s.p. state is being filled but other states are situated at larger energies and the BCS cannot produce an amount of pairing similar to that found in HFB.

The second general remark is about the fact that the $\langle(\Delta N)^2\rangle$ values obtained with our HFB+BCS approach are at most equal to those found in HFB. This is a clear indication that the HF+BCS pairing correlations are smaller than the HFB ones.

To test the validity of our HF+BCS model in the description of the density distributions, we have calculated elastic electron scattering cross sections and we have compared them with the experimental data when available. As example, in upper panels (a), (b) and (c) of Fig. 2 we show the charge densities obtained using the D1M interaction (red solid line) and the D1S one (black solid line) for the three nuclei ^{58}Ni, ^{116}Sn and ^{124}Sn, respectively. In panel (a) also the experimental charge density is shown. There are rather small differences between the results obtained with the two interactions.

Using the charge densities obtained with the D1M interaction, we plot in lower panels the elastic electron scattering cross sections (red solid lines) as a function of the scattering angle θ, compared to the experimental data (black full squares) taken from [9–11]. The incident electron energies are indicated in labels of the figure. In case of ^{58}Ni (panel (d)), an excellent agreement is found. For ^{116}Sn and ^{124}Sn nuclei (panels (e) and (f), respectively), the agreement with experiment is excellent for 330.0 MeV up to $\theta \sim 70^{\rm o}$, while for 500.0 MeV the results begin to be worse above $\theta \sim 50^{\rm o}$. This may be a consequence of the fact that the experimental charge density is not well reproduced with our model in the nuclear interior.

In conclusion, the HF+BCS approach here analyzed permits a feasible description of pairing effects. The validity of this approach has been validated comparing our results to those found in the more elaborated HFB calculations using the same interactions. Pairing correlations appear to be larger in HFB than in HF+BCS that points out a certain rigidity of BCS to mix various s.p. levels in the filling procedure.

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