Foundations of Quantum Mechanics

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The Lecce group concerned with the *Foundations of Quantum Mechanics* has worked on different but interconnected research topics.

Firstly, the research activity on the elaboration of a new theory (the *ESR model*) which avoids the problems and paradoxes of the standard interpretation of quantum mechanics (QM) has been continued. The ESR model embodies the mathematical formalism of QM into a broader mathematical framework, reinterpreting quantum probabilities as conditional on detection rather than absolute. Because of these features the ESR model avoids the basic problem of the quantum theory of measurement (the *objectification problem*), together with the related paradoxes (*Schrödinger's cat*, *Wigner's friend*, etc.). At the beginning of 2012 an article has been published which provides an improved version of the ESR model and compares the mathematical representations of proper and improper mixtures in this model with the mathematical representation of mixtures in QM. The article shows that the ESR model representation of improper mixtures coincides with the QM representation, while proper mixtures require an essentially different and new mathematical representation. On this basis the scheme of an experiment aiming to confirm or disprove the ESR model is proposed [1]. Furthermore, the Greenberger-Horne-Zeilinger (GHZ) experiment has been considered and described in the framework of the ESR model, showing that the toy models proposed by Szabó and Fine to supply a local interpretation of the GHZ experiment can be recovered as special cases within the local theory proposed by the Lecce group [2]. Finally, an article addressed to philosophers has been published which illustrated the unsolved problems of QM that have led the Lecce group to devise the ESR model and provides an informal description of the main features of this model [3].

Secondly, a research activity on the identification of quantum effects outside the microscopic domains has been continued and developed in collaboration with the Brussels research group.

(i) *Cognitive science.* Understanding the structure and dynamics of human concepts, how concepts combine and carry meaning in human thought is one of the major challenges in cognitive science. However, none of the existing theories on concepts explain 'how concepts combine'. This *combination problem* was revealed by Hampton's experiments (1988) which measured the deviation from classical (fuzzy) set theoretic membership weights of exemplars with respect to pairs of concepts and their conjunction or disjunction (*overextension*, *underextension*). The lack of a universally accepted theory for representing and structuring concepts has manifold consequences on other fields, such as psychology, artificial intelligence, linguistics, information retrieval. We have first explained how these findings in concept research lead one to recognize the need for quantum modeling [4,5]. Then, we have illustrated our *quantum modeling approach* by providing a description of the overextension for conjunctions of concepts measured by Hampton as an effect of quantum interference. Successively, we have considered a specific conceptual combination, we have performed a cognitive experiment based on this concept combination, and we have proved the presence of quantum entanglement in the given combination [4,6,7]. Finally, we have enlightened that also emergence occurs in conceptual processes, and that our quantum modeling approach applied in Fock space enables the description of emergence too [6,7]. We have also discussed some interesting applications of our *quantum cognition approach* to the study of the human brain [8].

(ii) *Economics.* In economics, the predominant model of decision making under uncertainty is *expected utility theory* (EUT). Notwithstanding its mathematical simplicity and predictive success, the structural validity of EUT at the individual level is questionable. Indeed, examples exist in the literature which show an inconsistency between agents' preferences and the predictions of EUT (*Ellsberg*, *Machina paradoxes*), which has strong implications on macroeconomic modeling, hence on the ability of economic theories to predict and avoid market crises. To overcome these difficulties a distinction between *risk* and *ambiguity* has been introduced in the literature which depends on the existence of a classical Kolmogorovian structure modeling these uncertainties. A careful analysis shows that context plays a relevant role in human decisions under uncertainty, and any probabilistic structure modeling contextual interactions between systems needs a non-Kolmogorovian framework endowed with a (generalized) quantum representation. We have thus proposed a notion of *contextual risk*, inspired by the probability structure of quantum theory, to mathematically capture situations in which ambiguity occurs [9–11]. The contextual risk approach has then been applied to the Ellsberg and Machina paradox situations, and a quantum mechanical representation in complex Hilbert space has been worked out which completely describes the ambiguity occurring in both situations [12]. Our contextual risk approach has finally been applied to stock price modeling in finance [13].

(iii) *Biology*. We have detected quantum–based structures in the study of ecological systems and their evolution. The dynamics of interacting species is mainly ruled by the Lotka-Volterra equations, which rest on classical physics and probability. But, for many interacting populations, these equations entail complex behavior, chaoticity and long-term unpredictability. For example, one cannot explain the coexistence of several phytoplankton species competing for the same limited resources (*paradox of the plankton*). If analyzed in detail, such plankton-type biodiversity arises from cyclic competition, an evolutionary analogue of the *scissors-paper-rock* (SPR) game. We have proved that an SPR game cannot be modeled in a single classical probabilistic framework but requires a non-commutative model of probability, such as those employed in quantum theory. Surprisingly, an SPR dynamics has been found in the mating competition of three 'side-blotched' lizard morphs. Therefore, we have used a set of experimental data collected on lizards from 1990 to 2011 to build an explicit quantum model in Hilbert space which faithfully accords with the collected probabilities [14].

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