Geometric aspects of symmetries and conservation laws in relativistic mechanics

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Tullio Levi-Civita (1873–1941) has been one of the most important mathematicians of the last century. Levi-Civita is best known for his work on the absolute differential calculus and because he was the founder with Gregorio Ricci-Curbastro of the subject now known as tensor analysis. Levi-Civita published nearly two hundred papers about classical mechanics, hydromechanics, thermodynamics, elasticity, the strength of materials, astronomy, electromagnetism, optics, relativity and quantum mechanics and pure mathematics. He was also the author of several treatises, and in collaboration with Ugo Amaldi he wrote an important reference book in classical mechanics: *Lezioni di Meccanica Razionale* [2]. This is a three volumes book first published in 1922 and then revised for the last time, after the death of Levi-Civita, by Amaldi in 1949.

In reading [2] it is impossible not to notice that this book does not contains any mention to the Noether's 1914 theorem [3]. For someone who knows the Levi-Civita scientific production it is hard to believe that the non-citation of Noether's theorem in [2] was possible. This fact is also clear from a bird's eye reading of [2]. The authors of this wonderful treatise consider in great detail, for a book in classical Mechanics, the theory of transformations, applying it to different topics. They also refer to the fundamental papers of Sophus Lie. Moreover, both Levi-Civita and Amaldi have been very active in studying the theory of transformation groups applied to various problems of mathematical physics. The journal where [3] is published is cited in the famous text [5], so it is plausible to think that Levi-Civita would read it more or less regularly.

We looked at the collected papers of Levi-Civita in an effort to try to understand possible reasons for this fact and we found a very interesting short note written at the onset of his career: the paper [**?**]. We decided to write an English translation of [**?**] because we thought that in this short note the connection between infinitesimal generators of symmetries (in the sense of Sophus Lie) and first integrals has been proposed for the first time in its general setting.

we are not claiming that in [**?**] you can find the two Noether's theorems from [3]. First of all Levi-Civita considers Hamiltonian finite dimensional systems and the connection between Lagrangian and Hamiltonian formulation was not rigorously stated. Moreover, Noether's results are much more general than what is contained in the short note by Levi-Civita. What we are claiming is that the connection between Lie's theory of groups and conserved quantities in mechanics was already clear in [**?**]. We point out that in [**?**] there are very few references, among which we found a citation to two short notes by Maurice Lévy and a short note of Valentino Cerruti. In those references, integrals of motion are in some sense connected with the symmetries of the mechanical system, but in an episodic and incomplete way.

We think that it is impossible to have a reasonable answer to our starting question: why are Noether theorems not cited in [2]? We believe that the general problems about the inception and the reception of the Noether theorems before 1950 in the case of Levi-Civita have to be summed up to the following specific facts:

- The connection between symmetries and constant of motions was already clear to Levi-Civita. This means that maybe Levi-Civita was not so surprised by the result of Noether and therefore maybe he was not so interested in reading the paper.

- Levi-Civita was interested more in the Hamiltonian formalism than in the Lagrangian one. Ugo Amaldi took the decision to stop his research activity soon in his career and therefore was no more up to date on the scientific literature to help Levi-Civita in discovering interesting papers. We recall that Olver says in [4] that: *The Hamiltonian version of Noether's theorem has a particularly attractive geometrical flavour, which remains somewhat masked in our previous Lagrangian framework*. We know that Levi-Civita was one of the first mathematicians interested in the geometry of Mechanics, and it is possible that he was

Lagrangean setting.

The second remark is about the fact that at least some papers by Levi–Civita seem to have not properly been noticed in the literature. Because of the racist laws introduced in Italy by the fascist regime, the notice of the death of Levi–Civita was only given by the Osservatore Romano and by foreign academies. For a long time a complete list of the publications by Levi–Civita was not available. Moreover, Ugo Amaldi's wish was that no obituary of him had to be diffused after his death. Therefore, it is not strange at all that the less famous papers of Levi–Civita have been forgotten for a long time.

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