Relative Helicity Phases in Planar Dirac Scattering

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The objective of this work is the derivation of the general formulas for spinor plane wave scattering from a stratified barrier potential [1]. The potential is assumed to be the zero component of a four vector potential, also known in the literature as an electrostatic potential. Our approach is three dimensional but the assumed stratified direction (x axis) of the potential together with the incoming three momentum determines the scattering plane within which lie both the reflected and transmitted momenta. We call this the x-y plane and hence, without lose of generality, we consider planar scattering, both diffusion and tunnelling.

Amongst the features that emerges for the reflected waves is the presence of both helicity *flip* and *non flip* terms. Furthermore, the corresponding probabilities depend explicitly upon the relative phase of the incoming helicity terms, when both are present. This allows one to *measure* this relative phase. As far as we know this has *not* been noted previously.

Consider, for the moment an incoming polarized beam say I_+, with $I_- = 0$. This could be achieved via a Stern-Gerlach apparatus. Then from our results the transmitted wave will also be polarized

$$T_+ = I_+ T \quad \text{and} \quad T_- = 0 \,,$$

but this will not be the case for the reflected wave [1], indeed

$$R_+ = I_+ R \quad \text{and} \quad R_- = I_+ \widetilde{R} \,.$$

However, these two helicity states will have a definite relative phase since $\widetilde{R}/R$ is imaginary. The exact value of this ratio depends upon momentum, but it does not depend upon the values of V_0 nor L, the barrier parameters. We thus have a means of producing reflected waves with known relative phase and calculable strengths.

Now, consider incoming waves, with a fixed relative phase. The above reflected waves provides an example. Let

$$I_+ = |\,I_+\,|\,e^{i\,\alpha} \quad \text{and} \quad I_- = |\,I_-\,|\,e^{i\,\beta} \,,$$

so that the relative phase of interest is $\alpha - \beta$. We subject this source to scattering by our barrier. Again the transmitted waves are of limited interest since they carry over the same relative phase between T_+ and T_-. On the other hand the situation is more complex for the reflected waves,

$$\begin{aligned} |\,R_+\,|^2 &= |\,I_+ R\,|^2 + |\,I_- \widetilde{R}\,|^2 - \\ &\quad 2\,|\,I_+ I_- R\,\widetilde{R}\,|\,\sin(\alpha-\beta)\,, \\ |\,R_-\,|^2 &= |\,I_- R\,|^2 + |\,I_+ \widetilde{R}\,|^2 + \\ &\quad 2\,|\,I_+ I_- R\,\widetilde{R}\,|\,\sin(\alpha-\beta)\,, \end{aligned} \tag{1}$$

which explicitly depend upon the relative phase. Thus, a second Stern-Gerlach apparatus followed by intensity (flux) measurement will yield the relative incoming phase.

Our knowledge of relative phases is almost non existent. For example, in a broad particle beam of constant density is the relative phase (when both helicities are present) the same for all particles or is it completely arbitrary? In the latter case the measurement of relative phase as described above would yield a null mean value. Nevertheless, even in this latter case, the reflection of a *polarized* beam will provide a fixed relative phase, for given incident angle and beam energy, for each individual particle in the beam.

We believe these predictions are worthy of *experimental verification.*

• The collaboration is now formalized as part of the interuniversity agreement between the State University of Campinas (Brazil) and the University of Salento (Lecce, Italy).

[1] S. De Leo and P. Rotelli, Phys. Rev. A **86**, 032113-5 (2012).

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