# Stressed Horizontal Convection

J. Hazewinkel, [1] F. Paparella [2] and W. R. Young[1]

[1]Scripps Institution of Oceanography, La Jolla, USA

[2]Dipartimento di Matematica e Fisica “Ennio De Giorgi”, Università del Salento, Italy

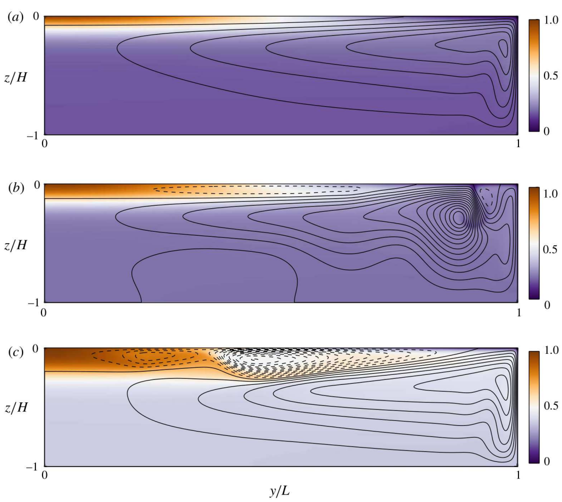


Figure 1. Buoyancy and streamfunction of three numerical solutions for progressively higher surface stress. The second and the third solution show regions of thermally indirect circulation.

The most common convective configuration is that where a fluid is confined between two plane, parallel, horizontal plates, kept at constant temperature. If the lower plate is sufficiently warmer than the upper plate the system is unstable to infinitesimal perturbations, and buoyancy changes induced by the temperature fluctuations induce convective motions into the fluid. This is the Rayleigh-Bnard set-up, and it is by far the most commonly studied case of convection.

However, most cases of convection in the real world are ill-approximated by the Rayleigh-Benard set-up. For example, the oceans are heated at the equator and cooled at the poles, but all heat exchanges happen at the surface of the water, which is to a very good approximation a surface of constant gravitational potential.

A convective set-up that heat and cools the working fluid at a surface of constant gravitational potential, in a container which, at the other boundaries is perfectly insulated, is called *horizontal convection*.

Although the horizontal convection set-up is perfectly able, at sufficiently high Rayleigh numbers, to trigger complicated, non-periodic, inpredictable motion in the fluid, it can never achieve a truly turbulent state, in the sense specified by Kolmogorov's theory of turbulence, because of the rigorous inequality that we proved in 2002:

$$\varepsilon < \kappa H^{-1} b_{max} \tag{1}$$

Here $\varepsilon$ is the kinetic energy dissipated by the fluid per unit time and mass, $\kappa$ is the thermal conduction coefficient of the fluid, $H$ is the depth of the container, and $b_{max}$ is the maximum buoyancy difference in the forcing boundary conditions. (Buoyancy is a convenient way to measure density fluctuations, and has the dimensions of an acceleration. Here $b_{max}$ plays the same role played by the plates temperature difference in Rayleigh-Bnard convection.)

Horizontal convection is therefore non-turbulent because if $H \to \infty$, or $\kappa \to 0$ (together with the fluid viscosity), then the kinetic energy dissipation $\varepsilon \to 0$, even if the forcing term $b_{max}$ is non-zero. In Kolmogorov's turbulence, on the other hand, the kinetic energy dissipation reaches a finite limit when, at constant forcing, the size of the domain is increased without bound, or, equivalently, the viscosity of the fluid is reduced to arbitrarily small values.

The non-turbulence of horizontal convection is more than a mere curiosity, because all the quantities in the inequality (1) are readily measured in the ocean. It turns out that the observed kinetic energy dissipation is orders of magnitude larger than what is allowed by (1). Therefore, the meridional overturning circulation of the world's ocean cannot be simply understood in terms of latitudinal differences of heating. Other factors must be at work, and they must either account for the near-totality of the energy budget of the ocean, or trigger an amplification of the meridional heat transport that would not be possible by convection alone.

In the most recent work we have explored the effect of a surface stress (produced by the wind, for example) on a horizontal convective set-up.

In an idealized geometry, we arrive at the exact result

$$H\varepsilon < \kappa\Delta\bar{b} + \overline{\boldsymbol{\tau}_s \cdot \boldsymbol{u}_s} \tag{2}$$

where $\Delta\bar{b}$ is the difference between the horizontally averaged buoyancy at the top and at the bottom of the container, $\boldsymbol{u}_s$ is the velocity of the fluid at the surface, and $\boldsymbol{\tau}_s$ is the mechanical stress externally imposed at the surface. It is then evident that, potentially, the action of the wind can bypass the severe constraint on kinetic energy dissipation imposed by (1).

We have further explored the problem by means of two-dimensional numerical simulations of the Boussinesq equations for convection. The interesting case is when the direction of the steady applied surface stress opposes the sense of the buoyancy driven flow. We obtain two-dimensional numerical solutions showing a regime in which there is an upper cell with thermally indirect circulation (buoyant fluid is pushed downwards by the applied stress and heavy fluid is elevated), and a second deep cell with thermally direct circulation. In this two-cell regime the driving mechanisms are competitive in the sense that neither dominates the flow. A scaling argument shows that this balance requires that surface stress vary as the horizontal Rayleigh number to the three-fifths power.

Modern descriptive studies emphasize that the Earths oceans have a multi-cell overturning structure, and that the shallow wind-driven cell, which has the greatest vertical temperature differences, is responsible for most of the heat transport (Talley 2003). We must cautiously interpret the oceanographic application of the very idealized problem of stressed horizontal convection. But we cannot resist remarking that the two-cell overturning pattern in our solutions is a feature of the ocean circulation, and probably for the same reason: stress forcing drives the shallow cell, while the deeper cell is associated with bottom-water formation and upwelling.

## REFERENCES

1. U. Frisch, Turbulence: the legacy of A. N. Kolmogorov, Cambridge University Press, Cambridge, 1995.
2. F. Paparella, W. R. Young, J. Fluid Mech. 466 (2002).
3. C. Wunsch, R. Ferrari, Annu. Rev. Fluid Mech. (2004).
4. L. D. Talley, J. Phys. Oceanogr., 33 (2003).
5. J. Hazewinkel, F. Paparella, W. R. Young, J. Fluid Mech. 692 (2012).