Clustering and Staircase Formation in Fingering Convection

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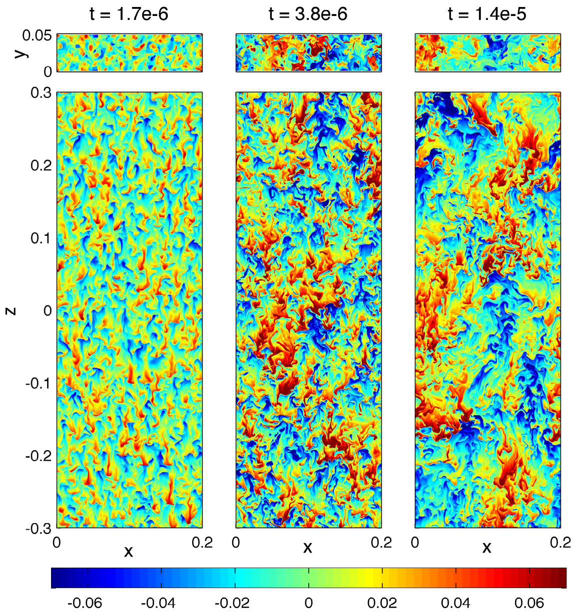

Figure 1. Horizontal (first row) and vertical (second row) sections of salinity at three different times in a simulation of finger convection carried out with unprecedented high resolution at the CASPUR supercomputer in Rome. The progressive clustering of fingers into larger scale structures is evident.

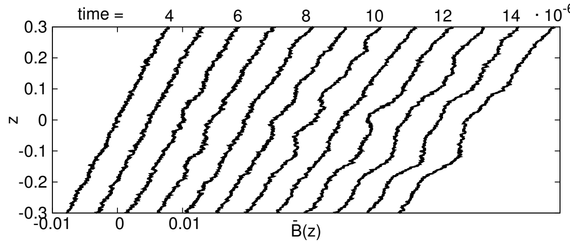

Figure 2. Time evolution of vertical profiles of horizontally averaged buoyancy, $\overline{B}(z)$, in the central portion of the domain. The frames after the first have been shifted to the right as a function of time.

Fingering convection plays an important role in many fields, ranging from stellar astrophysics to metallurgy, and is of particular interest in oceanography, where it may generate large heat and salt fluxes with an impact on the global circulation and on climate. This peculiar convective flow occurs when two buoyancy-changing scalars with different diffusivities, such as salt and temperature, make an overall bottom-heavy density stratification, but the least-diffusing one, taken alone, would produce a top-heavy stratification. In the fingering regime, a fluid parcel displaced from its equilibrium height exchanges the better diffusing and stabilizing scalar faster than it exchanges the destabilizing one, developing a buoyancy anomaly that further increases the displacement. The linear stability properties of the doubly diffusive instability are well understood and, close to marginality, tall, finger-like convection plumes emerge (hence the name). Far from the instability, in the highly nonlinear regime, these tall structures appear just as a short transient, after which the convection is sustained by the motion of almost spherical, blob-like structures, and the statistics of the fluctuations of the scalars become non-Gaussian.

An intriguing phenomenon that happens at high Rayleigh number and low density ratio (a measure of the relative contribution to the buoyancy of the two scalars) is an instability of the horizontally averaged profiles of temperature, salinity, and buoyancy. Laboratory experiments show that initially constant vertical gradients develop kinks which steepen and evolve into an alternation of well-mixed zones characterized by Bénard-like convection cells and high-gradient layers populated by fingers. Such staircase profiles are found in the main thermocline of the subtropical oceans and of many marginal seas.

We performed three-dimensional staircase-forming simulations with temperature and salinity held fixed at rigid top and bottom plates, in a regime of very low density ratio and high Rayleigh number, associated with very high numerical resolution (to our knowledge unprecedented for this problem). We find a previously unreported phenomenon of self-organization of fingers that clus-

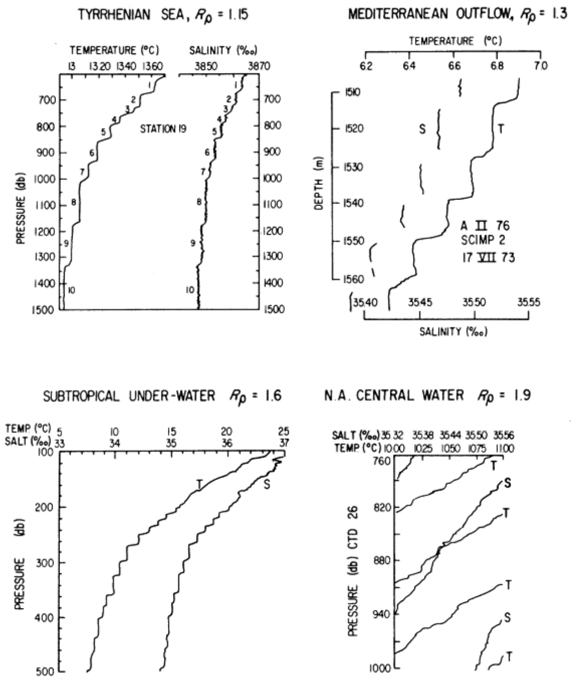

Figure 3. Four examples of real-world staircases produced by fingering convection in the oceans. Figure from [3].

ter together to form large-scale coherent structures.

We suggest a mechanism of staircase formation that occurs in two phases: first, the fingers group to form coherent structures at larger scales; then, the mechanical mixing induced by those clusters forms the staircases with a mechanism analogous to that of staircase formation in a stably stratified (nonconvective) stirred fluid.

We find evidence of a non-monotonic dependence the buoyancy flux F on the horizontally averaged buoyancy gradient $\overline{B}_z$. This is the effect of the interplay between the buoyancy transport operated by the fingers (which is up-gradient) and the transport caused by the stirring in the wake of the clusters (which is down-gradient). This evidence allows us to hypothesize the possibility of finding models of fingering convection having the structure of non-linear diffusion equations of the form

$$\frac{\partial \overline{B}}{\partial t} = -\frac{dF}{d\overline{B}_z}\frac{\partial^2 \overline{B}}{\partial z^2} + h.o.t.$$

where (so far unspecified) higher order terms are necessary to maintain the well-posedness of the problem when $dF/d\overline{B}_z > 0$ somewhere in the domain.

REFERENCES

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