A study on iterative reasoning

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This work [6] is in collaboration with researchers in experimental psychology.

The difficulty of reasoning iteratively has been observed mostly in game theory-based strategic games involving social interactions with other agents (such as the so called Beauty Contest Game) and has been partly attributed to bounded individual rationality. This difficulty has also been attributed to problems in adequately representing the beliefs, actions, social values and goals of other actors. We found that this same difficulty occurs consistently in various types of non-social tasks in which the source of the difficulty cannot be an inability to represent adequately other actors. Our findings are consistent with the hypothesis that an intrinsic difficulty in iterative reasoning originates in a tendency not to revise our initial mental representation of a problem in light of the initial conclusions that it implies.

People occasionally perform actions to obtain an initial predicted outcome without realizing that the effect may have additional foreseeable consequences. Examples of this behavior abound in interactive contexts such as strategic board games where one might miss the non-immediate but deterministic consequences of a move. Similarly, one rarely considers, say, that deciding to drive on a crowded road will make the road even more crowded and therefore slower . This tendency to underestimate the non-immediate consequences of one's own actions might originate from an inability to reason iteratively. In psychology iterative reasoning is defined as a reasoning strategy that can be adequately described by an iterative function.

An example of iterative task: the Beauty Contest Game

The boundaries of a person's ability to pursue a chain of iterative conclusions are illustrated by typical performance in guessing games such as the "Beauty Contest Game" (BCG), which was named after an example by John Maynard Keynes in 1936 and was first studied experimentally by Nagel [1]. In the BCG, N decision makers simultaneously choose a real number from the interval $I = [0, 100]$. The winner is the one whose number is closest to p times the mean of all chosen numbers (including his/her own), where $p \in (0, 1)$ is known.

The winner receives a prize, whereas the others earn nothing. In the case of a tie, the prize is divided equally. The game has a Nash equilibrium where all decision-makers choose 0. In the BCG a rational player will not choose either a random number or a favorite number or a number above $100p$. If a rational player believes that the other players are rational as well, he/she will not choose a number above $100\ p^2$. If he/she again believes that the others are also this rational, he/she will not pick a number above $100\ p^3$, and this pattern will continue until all of the numbers but zero have been eliminated. If all participants are rational and know that everybody else is rational then everybody should choose 0. However, the behavior of actual players is better described by a "what they think that I think that they think" style of iterated reasoning, that is a "iterated best reply" strategy. This strategy generates a cognitive hierarchy of thinking that allows to analyze the depth of the players' reasoning. In the k-level model ([1, 2]) this iterative choice (C) strategy for a BCG with a large N (and where each individual choice has a negligible effect on the aggregated mean) in recursive form is defined as follows:

$$C_0 = random(0,1) \qquad C_k = p\mu_{k-1} \qquad \mu_{k-1} = \frac{1}{N-1}\sum_{k-1}^{N-1} C_j$$

where the random function selects a number in the given range according to some probabilistic distribution, µ is the current choice temporarily attributed to the estimated mean of the other players' choices, and k index measures the players' iterative depth of thinking (i.e., how many iterative steps will the decision makers apply in choosing their numbers?) In level 0 reasoning, an individual considers only his or her beliefs and treats others as inert. In general, level k reasoning consists of attributing level $k-1$ reasoning to others. If the probability distribution of C_0 is uniform (with large N, this is a standard assumption of the k-level model), then $\mu_0 = 50$; it follows that $50p$ are level 1 choices,$50p^2$ are level 2 choices, and so on. In the general case, where N is not necessarily large ,t he player's rational choice at each iterative step is $C_k = p\mu_{k-1}\frac{N-1}{N-p}$ (for $N \to \infty$, $C_k = p\mu_{k-1}$, as in the large N case). This implies that in small-N versions of the BCG, the most rational C at each step is not $p\mu_{k-1}$. For example, if $p = .5$ and $N = 3$, in the first step a player might attribute randomness to the choice of the other two players (estimated $\mu_0 = 50$) and then opt for$C = 20$. At the second iterative step, he/she will attribute $\mu_1 = 20$ to the other players and then think that it would be better to choose $C = 8$; at the third step, he/she will choose $C = 3$. This process could continue until converging to 0. Choices in many Beauty Contest experimental games (e.g. [1]), but also in other games ([4, 2]), show limited steps of reasoning. First-round guesses are usually far from the equilibrium, either random choices (level 0), choices near $50p$ (level 1), or a few choices near $50p^2$ (level 2). The equilibrium 0 is chosen by less than 10% participants . The most impressive version of small-N BCG, where the participant's choice strongly affects the overall mean and thus the target number, is the 2-person version studied in [5]. Here a single step of rational reasoning suffices to establish that who picks the lowest number, wins (and thus, iteration is not required for choosing the weakly dominant choice, that is 0). However, most participants reasoned and chose like in $N > 2$ games, ignoring the strong influence of their own choice on the target number.

Explanations: rational vs. bounded rationality views

Previous literature has attributed poor performance in the BCG and other strategic games to a "lack of theory of mind", namely an inability at building models of the other players' thoughts and intentions. However, there are two different views of this inability. One is rational: players don't do the iterative process (or they limit their iterations) when they play strategic games because they do not believe that their opponents are capable of doing it. For example, for players that believe that all other players are level 0 thinkers, a level 1 choice is rational. The second interpretation implies bounded rationality in social contexts: players are inefficient at performing the iterative reasoning of the "what they think that I think that they think" variety that game theory assumes. The rational and the bounded rationality hypotheses are not mutually exclusive, and both are supported by empirical evidence (e.g. [5]). However, there is a third possibility, not implying lack of theory of mind. In recursion, as the number of steps increases so does the required amount of computational resources. Thus, persons might be intrinsically inefficient when performing any sort of iterative reasoning, whether it involves reasoning about others' thoughts and intentions, or reasoning in non-social situations. This "general bounded rationality" hypothesis is the main focus of the present study.

In this study, we investigated a variety of non-social tasks whose solution must/can be attained by iterative reasoning. We argued that if difficulties in iterative reasoning can be observed in tasks not involving interaction with other agents, then those difficulties could be attributed to individual cognitive constraints. By extension, similar difficulties might affect performance in interactive tasks such as the BCG.

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