A quantum operators approach to ecological models

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In recent years techniques from quantum mechanics theory have been applied to a number of classical problems (see e.g. [1–3]).

Here we refer to tools such as operator algebras and the number representation which were recently applied to problems in biology, sociology and economy [4–8].

In particular in [8] fermionic operators were used to describe interactions between two species in a patchy environment, taking into accounts also a migrations. More specifically two different cases were studied: a local situation in which two populations live together and are forced to interact and a situation in which the two species occupy different cells of a two–dimensional lattice, interact and move along the cells. Modeling the populations through fermionic operators implies that for each population there are two possible non-trivial situations. A *ground state* where there is a very low density, and an *excited state* in which the density is very high. Decreasing the density of the ground state annihilates the population; increasing the density of the excited state also leads to annihilation (the effect, for example, of a population too big for the carrying capacity of their neighbourhood). We cannot enter in much mathematical details here, in short this behaviour is explained by the defining properties of fermionic operators:

let $\mathcal{S}_1$ and $\mathcal{S}_2$ be two populations and associate to each of them an annihilation (a_j) and a creation operator ($a_j^\dagger$),defined on a Hilbert space $\mathcal{H}$ ($j = 1, 2$). Assume the following anticommutation rules:

$$\{a_i, a_j^\dagger\} = \delta_{i,j}, \qquad \{a_i, a_j\} = \{a_i^\dagger, a_j^\dagger\} = 0, \quad i, j = 1, 2,$$

where

$$\{x, y\} = xy + yx.$$

If $\varphi_{0,0}$ is the ground state, $a_1\varphi_{0,0} = a_2\varphi_{0,0} = 0$, the only non–trivial vectors of $\mathcal{H}$ ($dim(\mathcal{H}) = 4$) are

$$\varphi_{0,0} \qquad \varphi_{1,0} := a_1^\dagger\varphi_{0,0} \qquad \varphi_{0,1} = a_2^\dagger\varphi_{0,0} \qquad \varphi_{1,1} = a_1^\dagger a_2^\dagger\varphi_{0,0}.$$

Then define the number operators $\hat{n}_j := a_j^\dagger a_j$ and the associate eigenvalue equations:

$$\hat{n}_1\varphi_{n_1,n_2} = n_1\varphi_{n_1,n_2}, \qquad \hat{n}_2\varphi_{n_1,n_2} = n_2\varphi_{n_1,n_2}, \qquad j = 1, 2,$$

The *state of the system* is given as follows:

- $\varphi_{0,0}$: there are very few elements of both the populations;
- $\varphi_{1,0}$ ($\varphi_{0,1}$): there are very few elements of $\mathcal{S}_1$ ($\mathcal{S}_2$) but an abundance of $\mathcal{S}_2$ ($\mathcal{S}_1$);
- $\varphi_{1,1}$: both populations are abundant.

It is not possible increase the number of elements of, say, $\mathcal{S}_1$ decribed by $\varphi_{1,0}$ or $\varphi_{1,1}$, because this must be done applying the creation operator and it will destroys this population, since $(a_1^\dagger)^2 = 0$.

The dynamics of the system is described by a self-adjoint operator, the Hamiltonian.

We are now applying these tools to desertification dynamics, recently studied with different mathematical tools an described in the chapter on *Time fluctuations of vegetation patterns ...* in this report.

We use 3 populations whose dynamics is described by a fairly complicated Hamiltonian which takes into account various aspects of the problem, such as the effect of the interaction between populations and migrations (e.g. grazers moving in search of food, seeds dispersal).

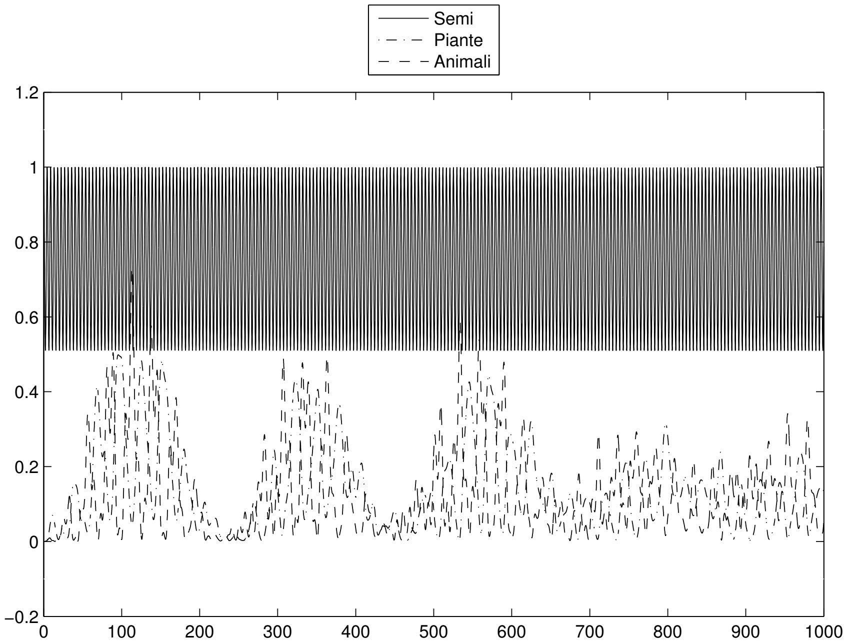

Figure 1. An example of the evolution of two populations in a cell

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