Some Anomalies of Farsighted Strategic Behavior

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Modern global communication and service infrastructures (e.g., Internet, P2P, wireless ad-hoc, social networks, etc.) are increasingly introducing decentralization, autonomy, and general lack of coordination among the heterogeneous network entities. The arising general mismatch between the system optimization goals and the competing users' private interests must be necessarily faced in emerging services and applications, and calls for a pressing solution of the resulting scientific and technological challenges. On this respect, useful tools and insights for modeling and analyzing the consequences of the autonomous users behavior on the system efficiency come from the integration of algorithmic ideas with techniques borrowed from Mathematical Economics and Game Theory. The fundamental approach adopted in the literature has been that of resorting on different concepts of equilibrium to characterize stable solutions consistent with the presence of rational and selfish users that have limited or no capabilities of cooperating, Nash equilibrium being among the most investigated one.

The central idea of quantifying the loss of efficiency deriving from non-cooperation is that of bounding the ratio between the social performance of the worst possible Nash equilibrium and that of the social optimal outcome, the so-called Price of Anarchy (PoA). Considerable research effort has been then devoted to bound the PoA in several non-cooperative games, including selfish routing, load balancing, linear congestion, fair cost sharing and consensus games.

Often Nash equilibria may not exist or it may be hard to compute them or the time for convergence to Nash equilibria may be extremely long, even if the players always choose a best-response move, i.e. a move providing them the smallest possible cost given the moves of the other players. Thus, recent research effort has focused on the evaluation of the performance after a limited number of selfish moves or a bounded number of rounds, with a round consisting of a sequence of best response moves, with each user moving exactly once.

As far as the specific games considered in this research are concerned, in cut games, Nash equilibria correspond to local optima of the classical local search algorithm. Moreover, a single round starting from the empty state in which every player has not selected any strategy yet coincides with a possible execution of the basic greedy algorithm. As a consequence, in both cases the price of anarchy is upper bounded by 2 and simple counterexamples show that this result is strict. Finally, the PoA after a single round starting from a generic state is $\Omega(n)$ [3]. In unrelated scheduling, it is known that the PoA of Nash equilibria (and thus of single rounds starting from a generic state) is unbounded, while it can be trivially verified that single rounds starting from the empty state have a PoA equal to the number of players n. Finally, the PoA of Nash equilibria in fair allocation games, that is cost sharing games based on the Shapley value in which the cost of each resource is equally split among the allocated players, again is exactly n, while in case of single rounds it is $O(\log^2 n)$ when starting from the empty state [2] and $n(n+1)/2$ when starting from a generic one [1].

One drawback of Nash equilibria and of the corresponding dynamics is that they often have disappointing performances: this has stimulated considerable research attempts in studying other reasonable solution concepts, like approximate and strong Nash equilibria, able to achieve better performances.

Equilibrium solutions alternative to Nash's one can be defined according to suitable extensions of the agents' rationality. In particular, considerable research effort focused on sequential games, modeling the rational behavior of agents who anticipate future strategic opportunities. More precisely, in the majority of equilibria notions, it is assumed that players simultaneously select their strategic choices. Even when speaking about speed of convergence and best response moves, the corresponding dynamics is actually the result of a myopic interaction among the players, in which each player merely selects the strategy being at the moment a good choice, without caring about the future evolutions of the game. In other words, the dynamics is not governed by farsighted strategic choices of the players. As a consequence, in the sequential setting, the Subgame Perfect Equilibrium (SPE) [6], better capturing the sequential rationality of the players, is the basic used equilibrium notion. Very recently, such equilibria have been investigated in the context of auctions [5], cut, consensus, unrelated scheduling and fair cost sharing allocation games [4]. A desired and expected effect of SPEs, remarked in [4], is that the corresponding farsighted choices may reduce the induced price of anarchy.

In [4], the authors considered sequential games and measured their Sequential Price of Anarchy (SPoA), that is, the price of anarchy of the corresponding Subgame Perfect Equilibria. In particular, they proved that the SPoA in cut games is between 2 and 4, while in unrelated machine scheduling it is at least $\Omega(n)$ and at most $m \cdot 2^n$, where n is the number of players and m the number of machines. Finally, for fair cost sharing allocation games, the authors just considered the singleton case in which each player can select a single resource and proved that under some restricted assumption the SPoA is exactly $H_n = \Theta(\log n)$.

Motivated by the above reasons, and by following the way marked out in [4], we consider some fundamental games in Algorithmic Game Theory. More precisely, we prove that the SPoA is exactly 3 in cut and consensus games. Moreover, we improve the previously known lower bound for unrelated scheduling to $2^{\Omega(\sqrt{n})}$ and refine the corresponding upper bound to 2^n. Finally, we determine essentially tight bounds for fair cost sharing games by proving that the SPoA is between $n + 1 - H_n$ and n. A surprising lower bound of $(n+1)/2$ is also determined for the singleton case.

Our results are quite interesting, counterintuitive and in some sense disappointing, as they put the expected performances of SPEs back in their right perspective. In fact, they show that farsighted choices may lead to a worse performance with respect to those yielded by a myopic behavior, as the price of anarchy of Nash equilibria and simple Nash rounds starting from the empty state happens to be lower than the SPoA in almost all of the considered games.

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