On the Sequential Price of Anarchy of Isolation Games

Anna Angelucci[a] Vittorio Bilò[b] Michele Flammini[a] and Luca Moscardelli[c]

[a]Dipartimento di Ingegneria e Scienze dell'Informazione e Matematica, Università di L'Aquila, Italy

[b]Dipartimento di Matematica e Fisica "Ennio De Giorgi", Università del Salento, Italy

[c]Dipartimento di Economia, Università di Chieti-Pescara, Italy

In competitive location games [2] players aim at choosing suitable locations or points in given metric spaces so as to maximize their utility or revenue. Depending on different parameters such as the underlying metric space, the number of players, the adopted solution concept, the customers' behavior and so on, several scenarios arise.

In this work we consider *isolation games* [6], a class of competitive location games in which the utility of a player is defined as a function of her distances from the other ones in an underlying edge-weighted graph. For example, one can define the utility of a player as being equal to the distance from the nearest one *(nearest-neighbor isolation game)*, or to the sum of the distances from all the other players *(total-distance isolation game)*.

Isolation games find a natural application in data clustering and geometric sampling. Moreover, as pointed out in [6], they can be used to obtain a good approximation of the strategy a player should compute in another competitive location game, called Voronoi game, which is among the most studied competitive location games. Here, the utility of a player is given by the total number of all points that are closer to her than to any other player (Voronoi area), where points which are equidistant to several players are evenly split up among them. As another interesting field of application for isolation games, consider the following problem in non-cooperative wireless network design: We are given a set of users who have to select a spot where to locate an antenna so as to transmit and receive a radio signal. When more than just one antenna is transmitting contemporaneously, interference between two or more signals may occur. If users are selfish players interested in minimizing the amount of interference their antenna will be subject to, they will decide to locate it as far as possible from the other ones thus giving rise to a particular isolation game.

The fundamental approach adopted in the literature in order to model the non-cooperative behavior of selfish players has been that of resorting on different concepts of equilibrium to characterize stable solutions, Nash Equilibrium being among the most investigated ones. In such a setting, the problem of measuring the loss of optimality due to the selfishness of the players becomes of crucial importance. To this aim, given a social function measuring the overall efficiency of any solution of the game, the notions of Price of Anarchy (PoA), obtained by comparing the social value of the worst Nash Equilibrium with that of the social optimum, and Price of Stability (PoS), obtained by comparing the social value of the best Nash Equilibrium with that of the social optimum, have been widely used in the Algorithmic Game Theory literature. Nevertheless, these two metrics naturally extends to other solution concepts alternative to Nash Equilibria.

These concepts can be defined according to suitable extensions of the agents' rationality. In such a setting, a recent research direction has focused on sequential games [1,3], modeling the strategic behavior of agents who anticipate future strategic opportunities. More precisely, in the majority of equilibria notions, it is assumed that players simultaneously select their strategic choices. Even when dealing with the speed of convergence and best-response moves, the corresponding dynamics is actually the result of a myopic interaction among the players, in which each player merely selects the strategy being at the moment a good choice, without caring about the future evolutions of the game. In other words, the dynamics is not governed by farsighted strategic choices of the players. As a consequence, in sequential games, the notion of Subgame Perfect Equilibrium (SPE) [5] is preferred to that of Nash Equilibrium since it better captures the rationality of farsighted players. Hence, the loss of optimality due to the presence of selfish players in sequential games is measured by means of the Sequential Price of Anarchy (SPoA), that is, the price of anarchy of Subgame Perfect Equilibria.

Motivated by the above reasons, we study the performance of Subgame Perfect Equilibria in sequential isolation games. More precisely, we focus on nearest-neighbor and total-distance sequential isolation games, and we show upper and lower bounds on the Sequential Price of Anarchy under the two social functions defined as the minimum utility per player (MIN) and the sum of the players' utilities (SUM).

For the basic case of $k = 2$ players, we prove that the Sequential Price of Anarchy is 1, i.e., every

Social Function	Nearest-Neighbor	Total-Distance
MIN	2	between $\frac{k-1}{k-2}$ and 8
SUM	∞ for $k \geq 4$ players between $\frac{3}{2}$ and 6 for $k = 3$ players	between $\frac{k(k-1)}{(k+1)(k-2)}$ and 3.765

Table 1
Sequential Price of Anarchy of isolation games for $k \geq 3$ players.

Subgame Perfect Equilibrium is an optimal solution, both for nearest-neighbor and total-distance sequential isolation games, under the two considered social functions. For the other cases ($k \geq 3$ players), the obtained results are summarized in Table 1. For nearest-neighbor games with SUM social function and $k \geq 4$ players, that is the case with an unbounded Sequential Price of Anarchy, we provide improved results for the special case of unweighted graphs by showing that the Sequential Price of Anarchy is between $4\frac{k-3}{k}$ and 8.

REFERENCES

1. V. Bilò, M. Flammini, G. Monaco, and L. Moscardelli. Some Anomalies of Farsighted Strategic Behavior. In *Proceedings of the 10th Workshop on Approximation and Online Algorithms (WAOA)*, 13–14 September, 2012, Ljubljana, Slovenia.
2. H. A. Eiselt, G. Laporte, and J. F. Thisse. Competitive Location Models: A Framework and Bibliography. *Transportation Science*, 27(1), 44–54, 1993.
3. R. Paes Leme, V. Syrgkanis, and É. Tardos. The Curse of Simultaneity. In *Proceedings of Innovations in Theoretical Computer Science (ITCS)*, ACM press, pp. 60–67, 2012.
4. R. Paes Leme, V. Syrgkanis and É. Tardos. Sequential Auctions and Externalities. In *Proceedings of the Twenty-Third Annual ACM-SIAM Symposium on Discrete Algorithms (SODA)*, SIAM, pp. 869–886, 2012.
5. M. J. Osborne and A. Rubinstein. A course in game theory. MIT Press, 1994.
6. Y. Zhao, W. Chen, and S. H. Teng. The Isolation Game: A Game of Distances. In *Proceedings of the 19th International Symposium on Algorithms and Computation (ISAAC)*, LNCS 5369, Springer, pp. 148–159, 2008.